MATH 2320 – Discrete Mathematics Assignment #3
Fall 2018

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday October 4th in Assignment Box #41.

1. Let $A = (-\infty, -5]$, $B = [-9, -\sqrt{2})$, $C = (0, \pi)$, and $U = \mathbb{R}$. What are:
   (a) $A \cap B$
   (b) $B \cup C$
   (c) $A^c \setminus (B \cap C)$
   (d) $(A \cup C) \setminus (A \cup B)^c$
   (e) $B \oplus C$
   (f) $C \setminus B^c$
2. Let A, B and C be subsets of some universal set U. Prove: $A \setminus (B \setminus C) = (A \setminus B) \cup (A \setminus C^c)$.
3. Consider the statement: $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$ for all sets A, B, C and D. Is this statement true? If yes, prove it; otherwise show that it is false.
4. Determine whether the relation $\mathcal{R}$ is reflexive:
   (a) $\mathcal{R} = \{(x, y) \in \mathbb{Z}^2 \mid x^2 + y^2 \text{ is odd}\}$
   (b) $\mathcal{R} = \{(x, y) \in \mathbb{Q}^2 \mid xy \geqslant 0\}$
5. Determine whether the relation $\mathcal{R}$ is symmetric:
   (a) $\mathcal{R} = \{(x, y) \in \mathbb{N}^2 \mid x + y = 9\}$
   (b) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x^2 - y^2 \geqslant 4\}$
6. Determine whether the relation $\mathcal{R}$ is antisymmetric:
   (a) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x \leqslant y\}$
   (b) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x^2 \leqslant y^2\}$
7. Determine whether the relation $\mathcal{R}$ is transitive:
   (a) $\mathcal{R} = \{(x, y) \in \mathbb{Z}^2 \mid x + y = 0\}$
   (b) $\mathcal{R} = \{(x, y) \in \mathbb{Q}^2 \mid x + y \in \mathbb{Z}\}$
8. Define the relation $\sim$ on $\mathbb{R}^2$ by $(a, b) \sim (c, d)$ if $2a - b = 2c - d$.
   (a) Prove that $\sim$ is an equivalence relation.
   (b) Provide a geometric description of $\overline{(2, 4)}$.
9. Define the relation $\sim$ on $\mathbb{Z}$ by $a \sim b$ if $3a + b$ is even.
   (a) Prove that $\sim$ is an equivalence relation.
   (b) What is $\overline{3}$?
   (c) What is $\mathbb{Z}/{\sim}$?