

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday September 27th in Assignment Box #41.

1. Let a, b, u and v be integers such that $u \neq 0$ and $v \neq 0$.
Consider the statement P : If $au + bv = 0$ then $a = b = 0$.

- (a) Is P true? If yes, then prove P ; otherwise show that P is false.
- (b) State the contrapositive of P .
- (c) State the converse of P .
- (d) State the negation of P .

2. Let a_1, a_2, a_3 be positive integers and let $m = \prod_{i=1}^3 a_i$.

Prove that at least one of a_1, a_2, a_3 is at least $\sqrt[3]{m}$.

3. Prove that $\log_2 5$ is irrational.
4. Determine whether the following statement is a tautology: P or $((P \text{ and } (\text{not } Q)) \Rightarrow R)$
5. Is the statement $P \Rightarrow (Q \text{ or } R)$ logically equivalent to the statement $(P \text{ and } (\text{not } Q)) \Rightarrow R$?
Explain why or why not.
6. Let $A = \{w, x, y, z\}$. List all of the subsets B of A such that

- (a) $\{x, y\} \subseteq B$
- (b) $\{x, y\} \not\subseteq B$
- (c) $\{x, y\} \subset B$
- (d) $B \subseteq \{x, y\}$
- (e) $B \not\subseteq \{x, y\}$
- (f) $B \not\subset \{x, y\}$

7. Let $A = \{1, 4, 7, 8\}$, $B = \{3, 6, 7, 9\}$, and $C = \{2, 4, 6, 7\}$.

- (a) Draw a Venn diagram showing the relationship between the sets, and where each element belongs.

- (b) What are:

- i. $A \cap B$
- ii. $B \cup C$
- iii. $A \cup (B \cap C)$
- iv. $(A \cup B) \cap C$
- v. $A \setminus (B \cap C)$
- vi. $(A \setminus B) \cap C$
- vii. $(B \cup C) \setminus A$
- viii. $\mathcal{P}(B)$

(over)

8. Let $A = \{a, b, \{a, b, c, d\}, \{a, d, e, g\}, f, \{c, f, g\}\}$.

(a) What is $|A|$?

(b) Indicate whether the following statements are true or false:

- i. $\emptyset \in A$
- ii. $f \in A$
- iii. $g \in A$
- iv. $\{f, g\} \in A$
- v. $\{f, g\} \subseteq A$
- vi. $\emptyset \subseteq A$
- vii. $f \subseteq A$
- viii. $\{a, b, c\} \subseteq A$
- ix. $\{a, b, c\} \in A$
- x. $\{b, f\} \subseteq A$
- xi. $\{b, f\} \in A$
- xii. $\{a, g\} \subseteq A$
- xiii. $\{a, g\} \in A$

9. Let A , B and C be sets. Prove: $(A \cap B) \times C = (A \times C) \cap (B \times C)$.