MATH 2320 – Discrete Mathematics — Assignment #2
Fall 2017

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday September 28th in Assignment Box #35.

1. Let a, b, u and v be integers such that $u \neq 0$ and $v \neq 0$.
   Consider the statement P: If $au + bv = 0$ then $a = b = 0$.
   (a) Is P true? If yes, then prove P; otherwise show that P is false.
   (b) State the contrapositive of P.
   (c) State the converse of P.
   (d) State the negation of P.

2. Let a_1, a_2, a_3 be positive integers and let $m = \prod_{i=1}^{3} a_i$.
   Prove that at least one of a_1, a_2, a_3 is at least $\sqrt[3]{m}$.

3. Prove that $\log_2 5$ is irrational.

4. Determine whether the following statement is a tautology: P or $\big((P \text{ and } (\text{not } Q)) \Rightarrow R\big)$

5. Is the statement $P \Rightarrow (Q \text{ or } R)$ logically equivalent to the statement $(P \text{ and } (\text{not } Q)) \Rightarrow R$? Explain why or why not.

6. Exercise 2.1.3 (except part (a)). This question is on page 42 of the textbook.

7. Let $A = \{1, 4, 6, 8\}$, $B = \{3, 7, 9\}$, and $C = \{2, 4, 6, 7\}$.
   (a) Draw a Venn diagram showing the relationship between the sets, and where each element belongs.
   (b) What are:
      i. $B \cup C$
      ii. $A \cup (B \cap C)$
      iii. $A \setminus (B \cap C)$
      iv. $(A \setminus B) \cap C$
      v. $(B \cup C) \setminus A$
      vi. $\mathcal{P}(B)$

(over)

8. Let $A = \big\{a, b, c, \{a, b, c, d\}, \{c, d, e\}, f, \{f, g\}\big\}$.

   (a) What is $|A|$?

   (b) Indicate whether the following statements are true or false:

      i. $\emptyset \in A$
      ii. $f \in A$
      iii. $g \in A$
      iv. $\{f, g\} \in A$
      v. $\{f, g\} \subseteq A$
      vi. $\emptyset \subseteq A$
      vii. $f \subseteq A$
      viii. $\{a, b, c\} \subseteq A$
      ix. $\{a, b, c\} \in A$
      x. $\{b, f\} \subseteq A$
      xi. $\{b, f\} \in A$

9. Let $A = (-\infty, -6)$, $B = (-8, 5)$, $C = [0, 12]$, and $U = \mathbb{R}$. What are:

   (a) $A \cap B$

   (b) $B \cup C$

   (c) $A^c \setminus (B \cap C)$

   (d) $(A \cup C) \setminus (A \cup B)^c$

   (e) $B \oplus C$

   (f) $C \setminus B^c$

10. Let A, B and C be sets. Prove: $(A \cap B) \times C = (A \times C) \cap (B \times C)$.