MATH 2050 – Linear Algebra I – Section 003   Assignment #8
Fall 2012

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Friday November 23rd in Assignment Box #42.

1. (a) Find an elementary matrix E such that $E \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 0 & \frac{1}{2} & 0 \end{bmatrix}$.

   (b) What is E^{-1}?

2. Express $L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$ as the product of elementary matrices and then use this factorisation of L to find L^{-1}.

3. Express $A = \begin{bmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & -5 & 2 \end{bmatrix}$ as the product of elementary matrices.

4. Let $A = \begin{bmatrix} 4 & 2 & 3 \\ 2 & 0 & 5 \\ 1 & 2 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$. Solve the equation $A\mathbf{x} = \mathbf{b}$ by finding an LU factorisation of A.

5. Let $A = \begin{bmatrix} -5 & 4 & 0 & 1 \\ -30 & 27 & 2 & 7 \\ 5 & 2 & 0 & 2 \\ 10 & 1 & -2 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -17 \\ -102 \\ -7 \\ -6 \end{bmatrix}$. Solve the equation $A\mathbf{x} = \mathbf{b}$ by finding an LU factorisation of A.

6. Consider the following system of equations:
$$\begin{array}{rcrcrcrcr} 6x_1 & - & 2x_2 & - & 4x_3 & + & 4x_4 & = & 2 \\ 3x_1 & - & 3x_2 & - & 6x_3 & + & x_4 & = & -4 \\ -12x_1 & + & 8x_2 & + & 21x_3 & - & 8x_4 & = & 8 \\ -6x_1 & & & - & 10x_3 & + & 7x_4 & = & -43 \end{array}$$

   (a) Express this system in the form of a matrix equation $A\mathbf{x} = \mathbf{b}$.

   (b) Find an LU factorisation of the matrix A.

   (c) Use the LU factorisation to find $\mathbf{x}$.

7. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 2 \\ 0 & 4 & 8 \\ 3 & -6 & 3 \\ 1 & 2 & 5 \end{bmatrix}$. Find a PLU factorisation of A.

8. For each matrix A below, find $\det(A)$ and if A is invertible then also find A^{-1}.

(a) $A = \begin{bmatrix} 6 & 4 \\ -3 & -2 \end{bmatrix}.$

(b) $A = \begin{bmatrix} -1 & 2 & 4 \\ 0 & 3 & 5 \\ 2 & -2 & 3 \end{bmatrix}.$

(c) $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & -1 & 3 \\ 4 & 7 & 5 \end{bmatrix}.$

9. Suppose that A is a 2×2 matrix with $\det(A) = 5$ and cofactor matrix $C = \begin{bmatrix} 3 & 1 \\ -2 & 1 \end{bmatrix}.$
   What is A?