MATH 2050 – Linear Algebra I – Section 003   Assignment #6
Fall 2012

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Wednesday October 31st in Assignment Box #42.

1. Use Gaussian elimination to find all solutions (if any exist) to the following system of linear equations:

$$
\begin{array}{rcrcrcl}
-2x_1 & - & 3x_2 & + & 2x_3 & = & 5 \\
5x_1 & - & 7x_2 & + & 3x_3 & = & 10 \\
2x_1 & + & x_2 & - & x_3 & = & 30
\end{array}
$$

2. Suppose that Alice purchased several bags of marbles at three different stores. Each bag bought at Sears contains 2 blue, 4 red and 7 yellow marbles. Each bag bought at Toys Я Us contains 3 blue, 3 red and 5 yellow marbles. And each bag bought at Zellers contains 4 blue, 2 red and 4 yellow marbles. Alice's collection of marbles contains 212 blue, 184 red and 333 yellow marbles.

   (a) State a system of linear equations, the solution to which reveals the number of bags of marbles that Alice bought at each store.

   (b) Express this system as a matrix equation.

   (c) Use Gaussian elimination to determine how many bags Alice bought at each store.

3. Use Gaussian elimination to find all solutions (if any exist) to the following system of linear equations:

$$
\begin{array}{rcrcrcrcr}
3x_1 & + & 2x_2 & - & x_3 & + & x_4 & = & 73 \\
4x_1 & - & 11x_2 & + & 2x_3 & - & 3x_4 & = & -242 \\
-x_1 & - & 6x_2 & + & 7x_3 & - & x_4 & = & -33 \\
2x_1 & + & 4x_2 & - & 6x_3 & - & 8x_4 & = & -786
\end{array}
$$

4. Consider the vectors $v_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 4 \\ -6 \\ 5 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$.

   (a) State a matrix equation, the solution to which reveals whether these vectors are linearly independent or linearly dependent.

   (b) Use Gaussian elimination to solve the equation of part (a) and hence determine whether these vectors are linearly independent or linearly dependent.

5. Use Gaussian elimination to find all solutions (if any exist) to the equation:

$$
\begin{bmatrix} -2 & 10 & 4 & 24 \\ -1 & 3 & 2 & 7 \\ -1 & 11 & 2 & 27 \\ 1 & 5 & -2 & 13 \end{bmatrix} x = \begin{bmatrix} 29 \\ 4 \\ 46 \\ 38 \end{bmatrix}.
$$

6. Consider the system of equations:

$$\begin{array}{rcrcrcr}
2x_1 & + & 4x_2 & + & 2x_3 & = & 3 \\
-x_1 & - & 3x_2 & - & x_3 & = & -5 \\
3x_1 & - & ax_2 & - & bx_3 & = & 15
\end{array}$$

Find conditions on a and b that:

(a) guarantee a unique solution for x_1, x_2 and x_3 (also state what this solution happens to be)

(b) assure that there are no solutions

(c) assure that there are infinitely many solutions