MATH2090 — MidTerm Fall 2017.

1. 1.

(a) For exact simple interest $t = 3 + \frac{29}{365}$ and

$$A(t) = 8000\left(1 + \frac{2.6}{100}t\right)$$

$$\therefore A\left(3\tfrac{29}{365}\right) \doteq 8640.526027$$

i.e. $8,640.53 (Nearest Cent)

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(b) Compound : $A(t) = \left(1 + \frac{2.6}{100}\right)^t 8000$

$$\therefore A(2) = (1.026)^2\, 8000$$

$$\doteq 8421.408$$

i.e. $8,421.41 (Nearest Cent).

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(c) Present Value at :– $P(t) = \dfrac{8000}{(1.26)^t}$, for $t = 19$

$$P(19) = \frac{8000}{(1.26)^{19}}$$

$$\doteq 4912.35938$$

i.e. $4,912.36 (Nearest Cent)

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(d) Interest is 2.6% nominal rate (converted quarterly), i.e.

$i^{(4)} = \frac{2.6}{100}$, so with time 3 years $= 3 \times 4 = 12$ quarters

the accumulated value is $8000 \times \left(1 + \frac{0.026}{4}\right)^{12} \doteq 8646.798483$

i.e. $8,646.80 (Nearest Cent)

Effective annual interest rate i given by

$$1 + i = \left(1 + \frac{0.026}{4}\right)^4$$

$$\doteq 1.026254$$

i.e. 2.63%.

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2. Start with the right side of the equation:—

$$\frac{s_{\overline{n}|} - a_{\overline{n}|}}{s_{\overline{n}|}\, a_{\overline{n}|}} = \frac{\left[\frac{(1+i)^n - 1}{i}\right] - \left[\frac{1-v^n}{i}\right]}{\left[\frac{(1+i)^n - 1}{i}\right]\cdot\left[\frac{1-v^n}{i}\right]}, \text{ from definitions}$$

$$= \frac{i\left\{\left[(1+i)^n - 1\right] - \left[1 - v^n\right]\right\}}{\left[(1+i)^n - 1\right](1-v^n)}$$

$$= \frac{i\left\{(1+i)^n + v^n - 2\right\}}{(1+i)^n - (1+i)^n v^n - 1 + v^n}, \text{ but as}$$

$(1+i)^n v^n = (1+i)^n \frac{1}{(1+i)^n} = 1$, we have

$(1+i)^n - (1+i)^n v^n - 1 + v^n = (1+i)^n + v^n - 2$ and so

12 $$\frac{s_{\overline{n}|} - a_{\overline{n}|}}{s_{\overline{n}|}\, a_{\overline{n}|}} = i\,\frac{\left\{(1+i)^n + v^n - 2\right\}}{\left\{(1+i)^n + v^n - 2\right\}} = i, \text{ as required.}$$

3. The account history gives three time periods to consider; let i_1, i_2 and i_3 be the corresponding rates of return.

3 Then, $$1 + i_1 = \frac{175{,}000}{150{,}000} = \frac{7}{6}$$

3 $$1 + i_2 = \frac{225{,}000}{175{,}000 + 25{,}000} = \frac{9}{8}$$

3 and $$1 + i_3 = \frac{210{,}000}{225{,}000 - 35{,}000} = \frac{21}{19}$$

Consequently the time weighted rate of return for the entire period, i_t, is given by

$$1 + i_t = (1+i_1)(1+i_2)(1+i_3)$$

$$= \frac{7}{6}\cdot\frac{9}{8}\cdot\frac{21}{19}$$

3 $$\doteq 1.450658, \text{ or } i_t \doteq 0.450658$$

1 i.e. the time-weighted rate of return is ~45.07%.

Q4 (a) The (present) value equation gives:

$$52000 = 12000\, a_{\overline{5}|} \quad \text{and as} \quad a_{\overline{5}|} = \frac{1-v^5}{i}, \text{ where}$$

$v = (1+i)^{-1}$; we have, after dividing by 12000,

$$\frac{13}{3} = \frac{1-v^5}{i}$$

Now put $x = 1+i$ (so $i = x-1$):

$$\frac{13}{3} = \frac{1-x^{-5}}{x-1}$$

i.e. $$\frac{13}{3} = \frac{x^5-1}{x^5(x-1)}$$

Cross-multiply to get:

$$13x^5(x-1) = 3(x^5-1)$$

i.e. $$13x^6 - (13+3)x^5 + 3 = 0$$

10 i.e. $$13x^6 - 16x^5 + 3 = 0, \text{ as required.}$$

(b) You need

3 >> roots([13 -16 0 0 0 0 3])

As $x = 1+i > 1$, look through the MATLAB output for the solution >1 (the real solution!) to find

$x = 1{\cdot}049677897870951$, as $x = 1+i$

i.e. $i \doteq 0{\cdot}0497$ (4 dec. places)

3 OR $4{\cdot}97\%$ annual interest.

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Q.5. If P is the amount invested then the present value equation $P = 2500\, a_{\overline{\infty}|}$ gives, as

$$a_{\overline{\infty}|} = \frac{1}{\left(\frac{2\cdot2}{100}\right)}, \text{ i.e. } a_{\overline{\infty}|} = \frac{1}{0{\cdot}022}$$

8 $\therefore P = 2500/0{\cdot}022 \doteq 113636{\cdot}363636$

i.e. \$113,636·36

Q.6. We need the effective monthly interest rate, j; we are given the nominal rate $i^{(2)} = \frac{4.8}{100}$.

3 We have $\left(1+j\right)^6 = \left(1 + \frac{4.8}{2\times 100}\right)$,

i.e. $j = (1.024)^{\frac{1}{6}} - 1$.

(a) Then the Present Value equation gives

$$250\,000 = X\, a_{\overline{12\times 20|}j}$$, where the monthly repayment is X dollars.

Now $a_{\overline{12\times 20|}j} = \dfrac{1-(1+j)^{-240}}{j} = \dfrac{1-(1.024)^{-40}}{(1.024)^{\frac{1}{6}}-1}$

$\doteqdot 154.7099900\,10765$, ~~so~~

$$X = \frac{250\,000}{a_{\overline{12\times 20|}j}} \doteqdot 1615.92\,667\,663\,287$$

5 i.e. the repayments are $1,615.93 (nearest cent).

(b) Accumulated Value $= X\, s_{\overline{12\times 20|}j}$, X as in (a).

$$= X \left[\frac{(1+j)^{40}-1}{j}\right]$$

$\doteqdot 645\,562.469521728$

6 i.e. $645,562.47

(c) Accumulated Value $= X\, s_{\overline{12\times 15|}j} \doteqdot 423,113.640352952$

3 i.e. $423,113.64. The outstanding debt is the compounded original debt minus the accumulated value of payments

6 i.e. $250000\,(1+j)^{12\times 15} - X\, s_{\overline{12\times 15|}j} \doteqdot 86145.353730\,6701$

28 i.e. $86,145.35 (Nearest Cent)