MATH2090 – Mathematics of Finance

Instructor: Dr. C. Radford

Mid-Term Test, FALL 2017

You may attempt all questions. The mark value of each question is indicated.
The total mark value of this test is 100.

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YOU MUST SHOW ADEQUATE WORKING WITH YOUR ANSWERS.

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No phones or notes allowed. Calculators permitted.
The examination time is 50 minutes

1. A bank account has a balance of \$8,000 on April 1, 2014. If there are no other withdrawals or deposits by the account holder answer the following questions.

   (a) Find the accumulated value at April 30, 2017, if we assume *exact simple interest* at 2.6% annually.

   (b) Find the accumulated value at March 31, 2016, if we assume *compound interest* at 2.6% annually.

   (c) Find the *Present Value* of the account on April 1, 1995 assuming *compound interest* at 2.6% annually.

   (d) Find the accumulated value of the account on March 31, 2017, if we assume compound interest with an annual interest rate of 2.6%, convertible quarterly. What is the effective annual interest rate in this case?

[26 Marks]

2. In our usual notation show that

$$i = \frac{s_{\overline{n}|} - a_{\overline{n}|}}{s_{\overline{n}|} a_{\overline{n}|}}.$$

[12 Marks]

3. An investment account shows the following activity:

   – March 31, 2012: a balance of \$150,000.
   – March 31, 2014: a balance of \$175,000.
   – April 1, 2014: a deposit of \$25,000.
   – March 31, 2016: a balance of \$225,000.
   – April 1, 2016: a withdrawl of \$35,000.
   – June 30, 2017: a balance of \$210,000.

   Find the *time-weighted* rate of return for the entire period covered in this account summary.

[12 Marks]

Questions 4, 5 and 6 over page

4. Parents set up a trust account of \$52,000 to fund their daughter's university studies over a five year period. The fund will make five annual payments of \$12,000 to cover their daughter's university expenses once she starts university in one year's time. What is the minimum annual interest rate, i, required to achieve the required annual payments.

   (a) Show that i satisfies the following equation,

   $$13X^6 - 16X^5 + 3 = 0, \text{ where } X = 1 + i.$$

   (b) What MATLAB command (or any alternative) would you use to numerically solve this polynomial equation to find X?
   The MATLAB output is

```
ans =

1.04967797870951 + 0i
0.999999999999999 + 0i
0.141244245497041 + 0.69435337804565i
0.141244245497041 - 0.69435337804565i
-0.55069861946718 + 0.36688931921682i
-0.55069861946718 - 0.36688931921682i
```

   What is the required annual interest rate?

[16 Marks]

5. A benefactor wishes to endow, in *perpetuity*, an annual scholarship of \$2,500. What is the minimum amount the benefactor must invest, at an annual interest rate of 2.2%, to fund the scholarship?

[8 Marks]

6. A home mortgage is advertised at an annual interest rate of 4.8%, converted semi-annually, with re-payments to be made at the end of each month over the 20 year period of the loan. Suppose I take \$250,000 mortgage under these conditions.

   (a) What are the monthly loan repayments?

   (b) What is the accumulated value of all the repayments at the end of the loan period?

   (c) Find the accumulated value of the repayments at the end of the 15th year and hence find the outstanding debt at the end of the 15th year?

[26 Marks]

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