MATH 2090

MidTerm Test Solutions, Fall 2016.

1. (a) Exact simple interest, time is $4 \times 365 + 29 = 1489$ days, so number of years is $\frac{1489}{365} = 4.0794521$

$$\text{Accumulated Value} = 5000.(1 + \tfrac{2.6}{100} \times 4.0794521)$$
$$= 5530.3288$$

ie. $5,530.33 (Nearest Cent)

(b)
$$\text{Accumulated Value} = 5000\left(1 + \tfrac{2.6}{100}\right)^4$$
$$= 5540.6338.$$

ie. $5,540.63 (Nearest Cent)

(c) Want present value of an account worth $5,000 in 17 years time, ~~so~~

$$\text{Present Value (1995)} = 5000 \times \frac{1}{\left(1 + \frac{2.6}{100}\right)^{17}}$$

$$= 3231.9518$$

ie. $3,231.95 (Nearest Cent).

(d) In this case 2.6% is a nominal interest rate (converted each quarter). So the accumulated value is:

$$\text{Accumulated Value} = 5000\left(1 + \tfrac{2.6}{4 \times 100}\right)^{4 \times 3}$$
$$= 5404.2491$$

ie. $5,404.25 (Nearest Cent)

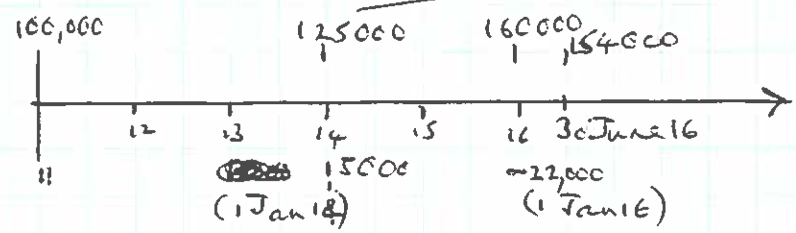
annual rate: $\left(1 + \frac{2.6}{4 \times 100}\right)^4 = 1.02625$
ie. 2.6255%

2.
$$\frac{a_{\overline{n}|}}{1 - i\,a_{\overline{n}|}} = \frac{\frac{1}{i}(1 - v^n)}{1 - (1 - v^n)} = \frac{(1 - v^n)}{i\,v^n}$$
$$= \frac{\left(\frac{1}{v^n} - 1\right)}{i}$$

i.e. $$\frac{a_{\overline{n}|}}{1-ia_{\overline{n}|}} = \frac{\left(\frac{1}{v^n}-1\right)}{i} = \frac{(1+i)^n-1}{i} = s_{\overline{n}|}$$

or $$s_{\overline{n}|} = \frac{a_{\overline{n}|}}{1-ia_{\overline{n}|}},$$ as required.

3.



(a) Time-weighted rate of return, i :—

$$1+i_1 = \frac{125000}{100000} = 1.25.$$

$$1+i_2 = \frac{160000}{125000+15000} = \frac{16}{14} \doteq 1.1428571.$$

$$1+i_3 = \frac{154000}{160000-22000} = \frac{154}{138} \doteq 1.115942.$$

Then $1+i = (1+i_1)(1+i_2)(1+i_3) \doteq 1.5942028$

∴ time-weighted rate of return is $i = 0.5942028$ or 59.42% (two decimal places).

(b) Dollar weighted rate of return, i :—

Equation of Value is :-

$$154000 = 100000(1+i)^{5.5} + 15000(1+i)^{2.5} - 22000(1+i)^{0.5}$$

i.e. $$1.54 = (1+i)^{\frac{11}{2}} + 0.15(1+i)^{\frac{5}{2}} - 0.22(1+i)^{\frac{1}{2}}$$

Put $X = (1+i)^{\frac{1}{2}} = \sqrt{1+i}$, ~~then~~

$$1.54 = X^{11} + 0.15X^5 - 0.22X$$

i.e. $$X^{11} + 0.15X^5 - 0.22X - 1.54 = 0.$$

Using MATLAB roots, and the fact that $X = \sqrt{1+i} > 1$

3/ we have $X = 1.0427301$

$\therefore \quad 1+i = X^2 = 1.0872861$

or $i = 0.0872861$

i.e. A rate of 8.73% (two places)

4\.

(a) The Present Value equation is

$$40000 = 20000\, a_{\overline{5|}}$$

$$\text{i.e.} \quad a_{\overline{5|}} = 2$$

$$\text{i.e.} \quad \frac{1-v^5}{i} = 2$$

$$\text{or} \quad 1 - v^5 = 2i$$

$$\text{i.e.} \quad 1 - \frac{1}{(1+i)^5} = 2i$$

Put $X = 1+i$, so $i = X-1$ and then

$$1 - \frac{1}{X^5} = 2(X-1)$$

$$\text{i.e.} \quad X^5 - 1 = 2X^5(X-1)$$

$$\text{or} \quad 2X^6 - 3X^5 + 1 = 0$$

(b) Using MATLAB and fact that $X = 1+i > 1$ we have $X \doteqdot 1.410415$ so $i = 0.410415$

an interest rate of 41.04% (two dec places)

5\. Present Value of the perpetuity is $1500\, a_{\overline{\infty|}} = \frac{1500}{i} = \frac{1500}{0.023}$

$$\text{i.e.} \quad \frac{1500}{0.023} = 65217.391$$

i.e. require \$65,217.39 (Nearest cent)

6. 5.6% is a nominal rate $\frac{5.6}{2\times100}$ is the 6-monthly rate

If the monthly rate is j then

$$(1+j)^6 = 1 + \frac{5.6}{2\times100} = 1.028$$

$$\text{i.e. } j = (1.028)^{\frac{1}{6}} - 1$$

(a) Present Value equation for monthly payments X:-

$$150\,000 = X\, a_{\overline{20\times12}|j}$$

$$\text{i.e. } X = \frac{150\,000}{a_{\overline{20\times12}|j}}$$

$$= \frac{150\,000}{\left[\dfrac{1-\dfrac{1}{(1.028)^{\frac{20\times12}{6}}}}{(1.028)^{\frac{1}{6}}-1}\right]}$$

$$\doteq 1034.8631$$

i.e. repayments are $\$1{,}034.86(31)$

(b) The outstanding debt just after (15×12)th payment is present value of the remaining $5\times12 = 60$ payments

$$\text{i.e. } 1034.86(31)\times a_{\overline{60}|j} = 54131.225.$$

i.e the remaining debt is $\$54{,}131.23$ (nearest cent)