MATH2090 – Mathematics of Finance

Instructor: Dr. C. Radford

Mid-Term Test, FALL 2016

You may attempt all questions. The mark value of each question is indicated.
The total mark value of this test is 100.

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YOU MUST SHOW ADEQUATE WORKING WITH YOUR ANSWERS.

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No phones or notes allowed. Calculators permitted.
The examination time is 50 minutes

1. A savings account has a balance of $5,000 on January 1, 2012. If there are no other withdrawals or deposits by the account holder answer the following questions.

   (a) Find the accumulated value at January 30, 2016, if we assume *exact simple interest* at 2.6% annually.

   (b) Find the accumulated value at January 1, 2016, if we assume *compound interest* at 2.6% annually.

   (c) Find the *Present Value* of the account on January 1, 1995 assuming *compound interest* at 2.6% annually.

   (d) Find the accumulated value of the account on January 1, 2015, if we assume compound interest with an annual interest rate of 2.6%, convertible quarterly. What is the effective annual interest rate?

[26 Marks]

2. In standard notation prove that

$$s_{\overline{n}|} = \frac{a_{\overline{n}|}}{1 - ia_{\overline{n}|}}.$$

[8 Marks]

3. An investment account shows the following activity:

   – December 31, 2010: a balance of $100,000.
   – December 31, 2013: a balance of $125,000.
   – January 1, 2014: a deposit of $15,000.
   – December 31, 2015: a balance of $160,000.
   – January 1, 2016: a withdrawl of $22,000.
   – June 30, 2016: a balance of $154,000.

Question 3 continued over page

(a) Find the *time-weighted* rate of return for the entire period covered in the account summary above.

(b) Show that the equation for the *dollar-weighted* rate of return (i.e. the effective annual compound interest rate, i) for the entire period can be written as,

$$X^{11} + 0.15X^5 - 0.22X - 1.54 = 0, \text{ where } X = \sqrt{1+i}.$$

Using the MATLAB roots function we can numerically solve for X, MATLAB gives

```
ans =

1.04273005356156 + 0i
0.891195981219006 + 0.577089686951494i
0.891195981219006 - 0.577089686951494i
0.414089344724339 + 0.951343787837139i
0.414089344724339 - 0.951343787837139i
-0.153935075906355 + 1.034307115045 88i
-0.153935075906355 - 1.034307115045 88i
-0.68903402019839 + 0.761355495826759i
-0.68903402019839 - 0.761355495826759i
-0.983681256619382 + 0.296470975244599i
-0.983681256619382 - 0.296470975244599i
```

Use this information to find the dollar-weighted rate of return, i.
[Recall: The '`i`' in the MATLAB output is $\sqrt{-1}$.] [24 Marks]

4. Joan sets up a trust fund with \$40,000 to fund her daughter's university studies over a five year period; Joan wants the fund to make five payments of \$20,000 (starting in one years time). We want to find the minimum annual interest rate, i, Joan requires to achieve the required annual payments.

(a) Show that i satisfies the following equation,

$$2X^6 - 3X^5 + 1 \text{ where } X = 1 + i.$$

(b) Using MATLAB to numerically solve this polynomial equation we get,

```
-->roots([2 -3 0 0 0 0 1])
ans =

1.410415
1.
0.1650226 + 0.7806076i
0.1650226 - 0.7806076i
- 0.6202301 + 0.4149757i
- 0.6202301 - 0.4149757i
```

What is the required interest rate?

[15 Marks] ***Questions 5 and 6 over page***

5. A benefactor wishes to endow, in *perpetuity*, an annual scholarship of $1,500. What is the minimum amount the benefactor must invest, at an annual interest rate of 2.3%, to fund the scholarship?

[5 Marks]

6. A home mortgage is advertised at an annual interest rate of 5.6%, converted semi-annually, with re-payments to be made at the end of each month over the 20 year period of the loan. Suppose I take $150,000 mortgage under these conditions.

   (a) What are the monthly loan repayments?

   (b) What is the outstanding debt on the loan after the last payment of the 15th year?

[22 Marks]