1

MATH 2090

Mid-Term Test — Fall 2015.

1.(a) Generally, $A(t) = P.(1+it)$;

with $i = \frac{3.4}{100}$ and $P = 10,000$ we have

$$A(t) = 10000(1+0.034 \times t)$$

For $t = 13$ (years)

$$A(13) = 10000(1+0.034 \times 13)$$
$$= 14420$$

Accumulated value is $\underline{\$14,420}$.

(b) Generally, $A(t) = P(1+i)^t$;

for $i = 0.034$ and $t = 13$

$$A(13) = 10000.(1+0.034)^{13}$$
$$= 15444.256$$

Accumulated value is $\underline{\$15,444.25(6)}$

(c) Generally, present value t years before accumulated value A is

$$P(t) = \frac{A}{(1+i)^t}$$

for $t = 4$ and $i = 0.034$

$$P(4) = \frac{10\,000}{(1.034)^4}$$

$$= 8748.1827$$

i.e. the present value in 1998 is $\underline{\$8,748.18(27)}$

(d) The effective monthly rate is $j = \frac{0.32}{100} = 0.0032$

Accumulated value after $t = 12 \times 13 = 156$ months is

$$A(156) = 10\,000\,(1+0.0032)^{156}$$
$$= 16460.903$$

i.e. a value of $\underline{\$16,460.90(3)}$.

(2

(e) Have a nominal annual rate converted quarterly, i.e. $i^{(4)} = \frac{3.4}{100} = 0.034$.

The accumulated value is

$$A(13) = 10000\left(1 + \frac{0.034}{4}\right)^{4\times 13}$$

$$= 15529.123$$

an accumulated value of $15,529.12(3)

The effective annual interest rate i is given by

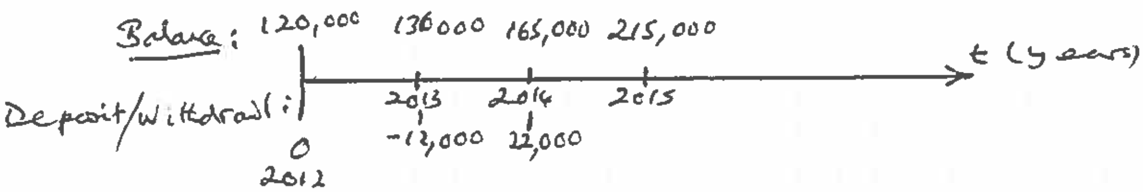
$$1+i = \left(1+\frac{0.034}{4}\right)^4$$

$$\doteqdot 1.034436, \text{ or}$$

$$i \doteqdot 0.034436$$

i.e. an effective annual rate of 3.44(36)%

2.



Effective interest rates for time intervals are

$$1+i_1 = \frac{136,000}{120,000} = \frac{136}{120} \doteqdot 1.133333$$

$$1+i_2 = \frac{165,000}{136,000 + (-12,000)} = \frac{165}{124} \doteqdot 1.3306452$$

$$1+i_3 = \frac{215,000}{165,000 + 22000} = \frac{215}{187} \doteqdot 1.1497326$$

And the time weighted rate of return, i_t, is given by :-

$$1+i_t = (1+i_1)(1+i_2)(1+i_3)$$

$$= \frac{136}{120} \times \frac{165}{124} \times \frac{215}{187}$$

$$\doteqdot 1.733871 \quad \text{or} \quad i_t \doteqdot 0.733871$$

i.e. a time weighted rate of return of 73.33(871)%

2 cf'd.

(b) The effective annual compound interest rate is i, so using our time line:—

$$\text{Accumulated value } 2015 = 215{,}000 = 120\,000 \times (1+i)^3 - 12\,000 \times (1+i)^2 + 22000 \times (1+i)^1$$

With $X = 1+i$ and rearranging:-

$$120X^3 - 12X^2 + 22X - 215 = 0.$$

From the given Scilab output we see there is only one real root greater than 1, so $X = 1.1971002$ and so $i = 0.1971002$, or a dollar weighted rate of return of $19.71(002)\%$.

3. Easiest method: $\ddot{s}_{\overline{n|}} = (1+i)^n \ddot{a}_{\overline{n|}}$

or $\dfrac{\ddot{s}_{\overline{n|}}}{\ddot{a}_{\overline{n|}}} = (1+i)^n$

But $\ddot{s}_{\overline{n|}} = \dfrac{(1+i)^n - 1}{d}$, so

$d\,\ddot{s}_{\overline{n|}} = (1+i)^n - 1$ and hence

$(1+i)^n = 1 + d\,\ddot{s}_{\overline{n|}}$

$$\therefore \frac{\ddot{s}_{\overline{n|}}}{\ddot{a}_{\overline{n|}}} = (1+i)^n = 1 + d\,\ddot{s}_{\overline{n|}} \text{ as required}$$

4. (a) The equation of value is (in this case the present value)

$$500\,000 = 100{,}000\, a_{\overline{10|}} \quad \text{with}$$

$$a_{\overline{10|}} = \frac{1-(1+i)^{-10}}{i} = \frac{1-x^{-10}}{x-1}$$

∴ (after dividing the equation by 100,000)

$$5 = \frac{1-x^{-10}}{x-1}$$

i.e. $5\times(x-1) = 1-x^{-10}$

so, $5x^{10}(x-1) = x^{10}-1$.

Rearranging :- $5x^{11} - 5x^{10} - x^{10} + 1 = 0$

or $5x^{11} - 6x^{10} + 1 = 0$, as required.

(b) aside: This is an 11^{th} degree polynomial, nine of the 12 terms have zero coefficients. So in SciLab first : a = zeros(1,12), then

a(1,1) = 5
a(1,2) = -6
a(1,12) = 1

to give a = [5 -6 0 0 ... 0 1]

To answer the question(!) ;—

Note there is just one real root strictly greater than 1, it is 1.1509841, so we have $x = 1.1509841$ so that

$$i = 0.1509841$$

i.e. an interest rate of 15.09(841)%.

5. Generally, the principle P is related to the payments X and interest rate by

$$P = \frac{X}{i}$$

for $X = 3500$ and $i = \frac{2.25}{100}$ we have

$$P = \frac{3500}{0.0225} = 155{,}555.56$$

i.e. the benefactor must invest \$155,555.56.

6. (a) Present value equation:-

$$(*) \quad 185000 = X\, a_{\overline{12\times 20}|\,j} \,, \quad j = \text{monthly rate.}$$

loan is for 12×20 months.

Interest rate $(1+j)^6 = 1 + \frac{4.3}{2\times 100}$

i.e. $1 + j = (1.0215)^{1/6}$

So from $(*)$

$$X = \frac{185000}{a_{\overline{12\times 20}|\,j}}$$

$$= \frac{185000 \times \left[(1.0215)^{\frac{1}{6}} - 1\right]}{1 - (1.0215)^{-240/6}}$$

$$\doteqdot 1146.766$$

i.e. the monthly repayments are \$1,146.76(6)

(b) The outstanding debt = Accrued Value of loan − total repayments.

i.e. Debt after 15 years $= 185000 \times (1.0215)^{\frac{12\times 15}{6}}$

$$- s_{\overline{12\times 15}|j}$$

$$= 185000 \times (1.0215)^{30} - \frac{[(1.0215)^{30} - 1] \times 1146.766}{(1.0215)^{\frac{1}{6}} - 1}$$

$$= 61870.37$$

i.e. a debt of \$61,870.37

[Alternatively :- this is just the present value of the debt at the end of the 15th year

i.e. $X\, a_{\overline{12(20-15)}|j}$]