# MATH2090 – Mathematics of Finance

Instructor: Dr. C. Radford

## Mid-Term Test, FALL 2015

**You may attempt all questions. The mark value of each question is indicated.**
**The total mark value of this test is 100.**

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**YOU MUST SHOW ADEQUATE WORKING WITH YOUR ANSWERS.**

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**No phones or notes allowed. Calculators permitted.**
**The examination time is 50 minutes**

1. A savings account has a balance of $10,000 on April 1, 2002. If there are no other withdrawals or deposits by the account holder answer the following questions.

   (a) Find the accumulated value at April 1, 2015, if we assume *simple interest* at 3.4% annually.

   (b) Find the accumulated value at April 1, 2015, if we assume *compound interest* at 3.4% annually.

   (c) Find the *present value* of the account on April 1, 1998 assuming *compound interest* at 3.4% annually.

   (d) Find the accumulated value of the account on April 1, 2015, if we assume compound interest with a monthly interest rate of 0.32%.

   (e) Find the accumulated value of the account on April 1, 2015, if we assume compound interest with a nominal annual interest rate of 3.4%, convertible quarterly. What is the effective annual interest rate?

[24 Marks]

2. An investment account shows the following activity:

   – August 31, 2012: a balance of $120,000.
   – August 31, 2013: a balance of $136,000.
   – September 1, 2013: withdrawal of $12,000.
   – August 31, 2014: a balance of $165,000.
   – September 1, 2014: deposit of $22,000.
   – August 31, 2015: a balance of $215,000.

   (a) Find the *time-weighted* rate of return for the entire period covered in the account summary above.

   (b) Show that the equation for the *dollar-weighted* rate of return (i.e. the effective annual compound interest rate, $i$) for the entire period can be written as,

$$120X^3 - 12X^2 + 22X - 215 = 0, \text{ where } X = (1+i).$$

***Question 2(b) continued over page***

Using SciLab we have

```
-->roots([120 -12 22 -215])
ans =

- 0.5485501 + 1.0935104i
- 0.5485501 - 1.0935104i
1.1971002
```

Use this information to find the dollar-weighted rate of return, $i$.
[Recall: The 'i' in the SciLab output is $\sqrt{-1}$.] [20 Marks]

3. Show that, in standard notation, $\dfrac{\ddot{s}_{\overline{n}|}}{\ddot{a}_{\overline{n}|}} = 1 + d\ddot{s}_{\overline{n}|}$, where $d = \dfrac{i}{1+i}$ the discount rate. [10 Marks]

4. A Company wishes to endow a university research grant of \$100,000 awarded at the end of each year for the next ten years. If the total sum of the original endowment is \$500,000 we want to find the minimum annual interest rate, $i$, which can achieve the required annual payouts.

   (a) Show that $i$ satisfies the following equation,

   $$5X^{11} - 6X^{10} + 1 = 0, \text{ where } X = 1 + i.$$

   (b) Using SciLab with `a = [5 -6 0 0 ··· 0 1]` to represent the polynomial we have,

   ```
   -->roots(a)
   ans =
   1.1509841
   1.
   0.6618634 + 0.5684244i
   ...
   - 0.7945452
   - 0.6565997 + 0.4534243i
   - 0.6565997 - 0.4534243i
   ```

   Where all the real roots are shown.
   What is the interest rate $i$?

   [18 Marks]

5. A benefactor wishes to endow, in *perpetuity*, an annual scholarship of \$3,500. What is the minimum amount the benefactor must invest, at an annual interest rate of 2.25%, to fund the scholarship? [8 Marks]

6. A home loan of \$185,000 is advertised at an interest rate of 4.3%, convertible semi-annually. The loan is repaid with payments made at the end of each month for the 20 year term of the loan.

   (a) What are the monthly loan repayments?

   (b) What is the outstanding debt on the loan after the last payment of the 15th year?

   [20 Marks]