Mid-Term Test, Fall 2014

1. (a) Accumulate Value :=

$$A(t) = 1000 \times \left(1 + \frac{2.4}{100}t\right), \text{ at time } t \text{ years.}$$

So we want

$$A(13) = 1000 \times (1 + 0.024 \times 13)$$
$$= 1312$$

i.e. the accumulated value is \$1,312. //

(b) Accumulated value at time t $\quad A(t) = 1000 \times \left(1 + \frac{2.4}{100}\right)^t$

we want

$$A(13) = 1000 \times (1.024)^{13}$$
$$\doteqdot 1361.1295$$

to the nearest cent this is \$1,361.13. //

2. (a) If i is the effective annual rate, the nominal rate is $i^{(12)} = \frac{4.1}{100}$ so $\left(1 + \frac{i^{(12)}}{12}\right)^{12} = 1 + i$

$$\text{i.e.} \quad \left(1 + \frac{4.1}{12 \times 100}\right)^{12} = 1 + i$$

$$\text{or,} \quad i = \left(1 + \frac{4.1}{1200}\right)^{12} - 1$$
$$\doteqdot 0.0417793$$

i.e. the effective annual rate is 4.17793 %. //

(b) $$\text{Present Value} = 10,000 \frac{1}{\left(1 + \frac{4.1}{12 \times 100}\right)^{12 \times 5}}$$
$$\doteqdot 8149.3201$$

i.e. the present value is \$8,149.32 (Nearest cent.) //

3. (a) Calculate the effective interest rate for each of the three time periods:

Dec 31, 2011 to Dec 31, 2012 $\quad 1 + i_1 = \dfrac{157,000}{150,000} \doteqdot 1.0466667$

Dec 31, 2012 to Dec 31, 2013 $\quad 1 + i_2 = \dfrac{165,000}{157,000 + (-12,000)} \doteqdot 1.137931$

Dec 31 2013 to June 30 2014 $\Big\}$ $1+i_3 = \dfrac{223000}{165,000+22,000}$

$$\doteqdot 1.1925134$$

So the time-weighted rate of return, i_t, is given by

$$1+i_t = (1+i_1)(1+i_2)(1+i_3) \doteqdot 1.4203245$$

$$\therefore i_t = 0.4203245$$

i.e. $42.03(245)\%$

(b) i is the effective annual interest rate so we can write the equation of accumulated value for the whole period, Dec 31, 2011 to June 30, 2014 (total of 2.5 years):-

$$223{,}000 = 150000 \times (1+i)^{2.5} - 12000 \times (1+i)^{1.5} + 22000 \times (1+i)^{0.5}$$

i.e. $$223 = 150(1+i)^{\frac{5}{2}} - 12(1+i)^{\frac{3}{2}} + 22(1+i)^{\frac{1}{2}}$$

with $x = (1+i)^{\frac{1}{2}}$ this is

$$223 = 150X^5 - 12X^3 + 22X$$

i.e. $\underline{150X^5 - 12X^3 + 22X - 223 = 0}$ as required.

Notice that $X = (1+i)^{\frac{1}{2}} > 1$ so using the Scilab output we see that $X \doteqdot 1.073891$ from which

$$1+i = X^2 \doteqdot 1.153419$$

$$\therefore i \doteqdot 0.153419$$

or an annual interest rate of 15.32% (two dec. places)

4. With interest rate i and $v = \dfrac{1}{1+i}$ we have

$$a_{\overline{n|}} = \frac{1-v^n}{i} \quad \text{and} \quad s_{\overline{n|}} = \frac{(1+i)^n - 1}{i}$$

$$\therefore \frac{s_{\overline{n|}}}{a_{\overline{n|}}} = \frac{(1+i)^n - 1}{1-v^n}$$

i.e. $$\frac{s_{\overline{n|}}}{a_{\overline{n|}}} = \frac{(1+i)^n - 1}{\left[1 - \frac{1}{(1+i)^n}\right]} = (1+i)^n \frac{\left[(1+i)^n - 1\right]}{\left[(1+i)^n - 1\right]}$$
$$= (1+i)^n$$

But $i\, s_{\overline{n|}} = (1+i)^n - 1$ so $(1+i)^n = i\, s_{\overline{n|}} + 1$

and $\therefore$ $\dfrac{s_{\overline{n|}}}{a_{\overline{n|}}} = 1 + i\, s_{\overline{n|}}$, as required. //

5. (a) Use the equation of present value :—

$$1\,000\,000 = 100\,000 \times a_{\overline{20|}}$$

i.e. $$10 = a_{\overline{20|}}$$

or $$\frac{1 - v^{20}}{i} = 10, \quad v = \frac{1}{1+i}.$$

i.e. $$1 - \frac{1}{(1+i)^{20}} = 10i$$

with $X = 1+i$ (so $i = X - 1$) we have

$$1 - \frac{1}{X^{20}} = 10(X-1)$$

$$\therefore \quad X^{20} - 1 = 10X^{20}(X-1)$$

So that, $10X^{21} - 11X^{20} + 1 = 0$. //

(b) $X = 1+i > 1$ so the SciLab output shows

$$X \doteq 1.0775469 \text{ ; so, as } i = X - 1,$$

$$i \doteq 0.0775469$$

i.e. we require an interest rate of

7.755 % (3 decimal places). //

6. Suppose the minimum invested amount is P dollars.

Then $$(1+i)P = P + 6000, \quad i = \frac{2.1}{100}$$

$$\text{i.e. } P = \frac{6000}{0.021} \doteqdot 285714.29$$

i.e. our benefactor must invest \$285,714.29. //

7. (a) The loan payments are monthly so we need the effective monthly interest rate i, say:—

The rate of 5.4% is the nominal annual rate convertible semi-annually, i.e. each 6 months:—

$$(1+i)^6 = 1 + \frac{5.4}{2 \times 100}$$

$$\text{or} \quad 1+i = \left(1 + \frac{5.4}{200}\right)^{\frac{1}{6}} \doteqdot 1.0044502$$

$$\text{i.e.} \quad i \doteqdot 0.004502 \; //$$

Now use the equation of present value with X the unknown monthly payments:—

$$170\,000 = X \times a_{\overline{12 \times 20|}}, \quad i \text{ as above.}$$

$$X = \frac{170\,000}{a_{\overline{240|}}}$$

$$= \frac{i}{1-(1+i)^{-240}} \times 170\,000$$

$$\doteqdot 1154.1216$$

i.e. Angela's monthly repayments are \$1,154.12(16). //

(b) The outstanding principal just after the last payment of the 12th year (i.e. just after the $12 \times 12 = 144$th payment) is $P_{144} = X\, a_{\overline{240-144|}} = X\, a_{\overline{96|}}$, X as in (a).

i.e. $P_{144} = 1154.1216 \times \frac{[1 - v^{96}]}{i}$

$$= 1154.1216 \times \frac{\left[1 - \left(1 + \frac{5.4}{200}\right)^{\left(-\frac{96}{6}\right)}\right]}{\left(1 + \frac{5.4}{200}\right)^{\frac{1}{6}} - 1}$$

$$\doteqdot 90007.319$$

The outstanding principal is \$90,007.31(9)//.

(c) Angela now has loan of \$90,007.319 and the new monthly rate is given by

N.B. Question was meant to have 5% semi annual

$$(1+i)^6 = 1 + \frac{5}{2 \times 100} \quad \text{or} \quad 1 + i = \left(1 + \frac{5}{200}\right)^{\frac{1}{6}}$$

and the term of the loan is 8 years with $8 \times 12 = 96$ payments.

Angela's new payments X are given by

$$90\,007.319 = X\, a_{\overline{8 \times 12|}}, \; i = \left(1 + \frac{5}{200}\right)^{\frac{1}{6}} - 1$$

$$\therefore \quad X \doteqdot 90007.319 \times \frac{\left[1 - \left(1 + \frac{5}{200}\right)^{\left(-\frac{96}{6}\right)}\right]}{\left(1 + \frac{5}{200}\right)^{\frac{1}{6}} - 1}$$

$$\doteqdot 1137.2884 \; //$$

i.e. Angela's new payments are \$1,137.28(84)//.

Angela's ~~?~~ saving over the eight years of the new loan will be:—

$$\{\text{Difference in monthly payments}\} \times \overbrace{8 \times 12}^{\text{Number of payments}}$$

$$= \{1154.1216 - 1137.2884\} \times 8 \times 12$$

$$= 1615.9872$$

i.e. Angela ~~saves~~, to the nearest cent, \$1,615.99//.