

MATH2090 – Mathematics of Finance

Instructor: Dr. C. Radford

Mid-Term Test, FALL 2014

You may attempt all questions. The mark value of each question is indicated.
The total mark value of this test is 100.

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YOU MUST SHOW ADEQUATE WORKING WITH YOUR ANSWERS.

1. A bank account has a balance of \$1,000.00 on January 1, 2001.
 - (a) Find the accumulated value at January 1, 2014, if we assume *simple interest* at 2.4% annually.
 - (b) Find the accumulated value at January 1, 2014, if we assume *compound interest* at 2.4% annually.

[12 Marks]

2. A finance company offers an investment opportunity at an interest rate of 4.1% convertible monthly.
 - (a) What is the effective annual interest rate for this investment?
 - (b) What is the *present value* of an investment worth \$10,000 in five years time?

[12 Marks]

3. An investment account with a bank shows the following activity:
 - December 31, 2011: a balance of \$150,000.
 - December 31, 2012: a balance of \$157,000.
 - January 1, 2013: withdrawal of \$12,000.
 - December 31, 2013: a balance of \$165,000.
 - January 1, 2014: deposit of \$22,000.
 - June 30, 2014: a balance of \$223,000.
 - (a) Find the *time-weighted* rate of return for the entire period covered in the account summary above.
 - (b) Show that the equation for the *dollar-weighted* rate of return (i.e. the effective annual compound interest rate, i) for the entire period can be written as,

$$150X^5 - 12X^3 + 22X - 223 = 0, \text{ where } X = (1 + i)^{\frac{1}{2}}.$$

[Question 3 continued over page.]

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Using SciLab we have
-->roots([150 0 -12 0 22 -223])
ans =
- 0.8949612 + 0.6489521i
- 0.8949612 - 0.6489521i
1.073891
0.3580157 + 1.0023037i
0.3580157 - 1.0023037i

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Use this information to find the dollar-weighted rate of return, i .

[Recall: The 'i' in the SciLab output is $\sqrt{-1}$.] [20 Marks]

4. Show that, in standard notation, $\frac{s_{\overline{n}|}}{a_{\overline{n}|}} = 1 + is_{\overline{n}|}$. [8 Marks]

5. A court settlement orders that Mr. Jones be paid an annuity of \$100,000 at the end of each year for the next twenty years. If the total sum originally awarded was \$1,000,000 we want to find the minimum annual interest rate, i , which can achieve the required annual payouts.

(a) Show that i satisfies the following equation,

$$10X^{21} - 11X^{20} + 1 = 0, \text{ where } X = 1 + i.$$

(b) Using SciLab with $\mathbf{a} = [10 \ -11 \ 0 \ 0 \ \dots \ 0 \ 1]$ to represent the polynomial we have,

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-->roots(a)
ans =
1.0775469
1.
0.8721991 + 0.3308388i
...
- 0.8617238
- 0.8217956 + 0.2609011i
...

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Where all the real roots are shown.

What is the interest rate i ?

[18 Marks]

6. A benefactor wishes to endow, in *perpetuity*, an annual scholarship of \$6,000. What is the minimum amount the benefactor must invest, at an annual interest rate of 2.1%, to fund the scholarship?

[10 Marks]

7. Angela takes out a home loan of \$170,000 at an interest rate of 5.4%, convertible semi-annually. The loan is repaid with payments made at the end of each month for the 20 year term of the loan.

(a) What are Angela's monthly loan payments?

(b) Find the outstanding principal on the loan just after the last payment made in the 12th year of the loan.

(c) Angela refinances her loan at the end of the 12th year, her new interest rate is 5%, convertible semi-annually, over an eight year term with equal payments made at the end of each month. What are Angela's new monthly payments? How much will Angela save by refinancing her loan in this way?

[20 Marks]