

Memorial University of Newfoundland
Department of Mathematics and Statistics

Mathematics 2090

Final Examination

Fall 2013

Instructor: M. S. Grewal

December 10, 2013

9:00 AM – 11:00 AM

Room: SN 4083

Student's Last Name-----

First Name-----

Student Number-----

INSTRUCTIONS:

- **The examination consists of 10 questions on 10 pages.**
- **Total mark for the examination is 50.**
- **Formulae sheet and calculator are permitted.**
- **Relax! Good Luck!**

1. (a) If a discounted loan is offered at a rate of discount $d = 8\%$, what is the equivalent rate of interest i ? [1]

(b) What nominal rate of interest, compounded monthly, is equivalent to $i^{(12)} = 9.5\%$? [1]

(c) A mutual fund deposit of \$1000 increases in value by \$560 over 30 months. Determine the continuous rate of increase δ . [2]

2. Prove each of the following identities mathematically.

a. $d \left(1 + \frac{i}{2}\right) = i \left(1 - \frac{d}{2}\right)$ [1]

b. $1 - d = (1 + i)^{-1} = v$ [1]

c. Find n such that $1 + \frac{i^{(n)}}{n} = \frac{1 + \frac{i^{(10)}}{10}}{1 + \frac{i^{(12)}}{12}}$ [2]

3. Payments of \$100, \$200, and \$500 are paid at the end of years 2, 3, and 8, respectively. If $i^{(1)} = 5\%$ per annum effective, determine the time, t , at which a single payment of \$800 would be equivalent. [3]

4. How long should \$5000 be left to accumulate at $i = 0.10$ in order that it may amount to twice the accumulated value of another \$5000 deposited at the same time at 8% effective? [4]

5. Melissa borrows \$5000 on January 1, 2000, and another \$5000 on January 1, 2003. She repays \$3000 on January 1, 2002, and then finishes repaying his loan by paying \$10,000 on January 1, 2005. What effective annual rate of interest is Melissa being charged? [6]

6. An alumna gives a cash donation of \$100,000 to her former university. The donation is invested in an endowment fund at $i^{(1)} = 7\%$. An annual scholarship is to be paid from this fund of $\$R$ for the first 10 years, followed by $\$1.5R$ thereafter. What is R ? [5]

7. An apartment building consists of 146 apartment units of equal size. The board of directors of Homeowners' Association estimated that the building will need new carpeting in the halls at a cost of \$78,000 in five years. Assuming that the association can invest its money at $i^{(12)} = 5.4\%$, what should be the monthly sinking-fund assessment per unit? Also, show the first two lines and the last two lines of the sinking-fund schedule. [5,3]

8. A \$5000 bond maturing at 105 on September 1, 2031, pays interest at 7% per year convertible semi-annually. Determine the purchase price on March 1, 2010 to guarantee a yield of $i^{(2)} = 6.8\%$. Construct the $t = 8$ and $t = 15$ rows of the bond amortization schedule. [6]

9. A 10-year \$1000 face value bond, redeemable at par, earns interest at 9% convertible semi-annually. The bond is bought to yield 8% convertible semi-annually. Find the book and market values 4 months after the 7th coupon has been paid. [6]

10. A \$1000 loan is repaid by annual payments of \$150, plus a smaller final payment. If $i = 0.11$, and the first payment is made one year after the time of the loan, find the amount of principal and interest contained in the third payment. [4]

Formulae Sheet

$$1 + i = \left[1 + \frac{i^{(m)}}{m}\right]^m$$

$$1 - d = \left[1 - \frac{d^{(m)}}{m}\right]^m$$

$$e^{\delta} = 1 + i$$

$$i_t = \frac{B_t}{B_{t-1} + W_{t-1}} - 1$$

$$i = (1 + i_1)(1 + i_2)\dots(1 + i_n) - 1$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$a_{\overline{n}|} = \frac{1 - v^n}{i}$$

$$S_{\overline{n}|} = \frac{(1 + i)^n - 1}{i}$$

$$\ddot{a}_{\overline{n}|} = a_n(1 + i)$$

$$\ddot{S}_{\overline{n}|} = \frac{(1 + i)^n - 1}{d} = \ddot{a}_n(1 + i)^n$$

$$a_{\infty|} = \frac{1}{i}$$

$$\ddot{a}_{\infty|} = \frac{1}{d}$$

$$i a_{\overline{n-t}|} = 1 - v^{n-t}$$

$$P = (Fr)a_{\overline{n}|}i + C(1 + i)^{-n}$$

$$B_t = (Fr)a_{\overline{n-t}|} + Cv^{n-t}$$

$$P/D = C + (Fr - Ci)a_{\overline{n}|}$$

$$i = \frac{Fr - (Q - C)/n}{1/2(Q + C)}$$