MATH2090 – Mathematics of Finance

Assignment 9

Name:

MUN Number:

Due Date: Monday, 20 November

1. Consider our simplified market model with two assets, $S(t)$ and $B(t)$, a stock and a bond, and two possible times $t = 0, 1$; with the bond has a price of $B(0) = 100$ dollars, and bond yield rate of $i_B = 10\%$; with a stock price today of $S(0) = 50$ dollars.
   If my portfolio consists of 100 shares and 40 bonds, and that the possible share prices at $t = 1$ are,
   $$S(1) = \begin{cases} 80, & \text{with probability } 0.75; \\ 40, & \text{with probability } 0.25. \end{cases}$$
   (a) What is the initial value $V(0)$ of the portfolio? What are the possible values at $t = 1$?
   (b) Calculate the possible yields (or returns), i_V, of the portfolio.
   (c) What is the *expected return* on the portfolio?
   (d) What is the *risk*, as measured by the standard deviation, of this portfolio?

2. Consider a market with one stock and one bond type with the following price structure,
   $$B(0) = 100,\ B(1) = 105;\ S(0) = 100 \text{ and}$$
   $$S(1) = \begin{cases} 110, & \text{with probability } 0.75; \\ 90, & \text{with probability } 0.25. \end{cases}$$
   Bob establishes the portfolio $(150, 150)$ and Sally the portfolio $(100, 200)$.
   (a) Show that Bob's and Sally's portfolios have the same initial cost and that they both have the same expected yield.
   (b) Using the standard deviation of the portfolio yield compare the risk of the two portfolios. From (a) the two portfolios would appear very similar, in risk terms which portfolio is to be preferred?
   (c) In fact show that the portfolio $(0, 300)$, satisfies the same conditions, (a), as Bob's and Sally's and is in fact risk free in terms of its assets and in terms of the standard deviation of the yield.

3. Consider a random variable X with just two possible values X_1 and X_2, occurring with probabilities p_1 and p_2, respectively. Consider the random variable $Y = \alpha X + \beta$, where α and β are fixed. Show that the expectation of Y is given by,
   $$E(Y) = \alpha E(X) + \beta.$$
   Of course this formula is easily generalized to any random variable X.
   Hence show that for our portfolio (x, y), with $V(t) = xS(t) + yB(t)$, the expectation of the yield can be written as
   $$E(i_V) = \frac{xS(0)E(i_S) + yB(0)i_B}{xS(0) + yB(0)}.$$