

# MATH2090 – Mathematics of Finance

## Assignment 5

Name:

MUN Number:

**Due Date:** Monday, 23 October

1. Consider an annuity with  $n$  payments made at the end of each for  $n$  years with an annual interest rate of  $i$ . Suppose also that the payments form a geometric sequence,

$$P_k = Pr^{k-1}, \text{ for } k = 1, \dots, n \text{ with } P \text{ and } r \text{ fixed constants.}$$

Show that the present value of the annuity (at  $t = 0$ ) is given by,

$$P.V. = P\nu \left[ \frac{1 - (r\nu)^n}{1 - r\nu} \right], \text{ where, as usual, } \nu = \frac{1}{1 + i}.$$

What is the accumulated value, A.V., just after the last payment?

2. A court awards a settlement of \$1,500,000 in the following terms:
- The money is to be invested with (minimum) annual interest rate of 2.7%;
  - the award is to be paid with the initial payment one year after the settlement date and the remaining payments made at the end of each subsequent year for 20 years; the payments are to increase in those subsequent years in line with an inflation rate of 2.1% per annum.
- (a) What is the amount of the first payment?
- (b) What is the amount of the last payment?
3. Jean takes a mortgage of \$167,000 with her bank at an interest rate of 4.6%, convertible semi-annually, and over a term of 25 years. Jean repays the mortgage with equal payments made at the end of each month.
- (a) What are Jean's monthly payments on her mortgage?
- (b) What is the outstanding principal on Jean's loan immediately after the last payment in the 20th year?
- (c) What portion of the last payment in the 20th year is interest and what portion is principal?
- (d) Jean refinances her loan, immediately after her last payment in the 20th year: Jean pays a lump sum of \$10,000, and then repays the remaining debt over the 10 years with an annual interest rate of 4.1%, and equal payments made at the end of each month. What are Jean's new monthly payments?