MATH2090 – Mathematics of Finance

Assignment 10

Name:

MUN Number:

Due Date: Monday, 27 November

1. Consider a portfolio $(20, 10, 10)$ consisting of shares, price $S(t)$, and bonds, value $B(t)$, where $S(0) = 50$, $B(0) = 100$ and $B(1) = 110$, and long forward contracts at price F. Suppose the possible future prices of the shares is given by,

$$S(1) = \begin{cases} 55, & \text{with probability } 0.75; \\ 46, & \text{with probability } 0.25. \end{cases}$$

   (a) Calculate the value of the forward price F.

   (b) Calculate the possible yields of this portfolio and hence find the expected yield.

   (c) Find the risk as measured by the standard deviation of the yield.

2. Consider a market with one stock $S(t)$, one bond type $B(t)$ and one call option $C(t)$ with strike price P_c, all with the following pricing structure,

$$P_c = 46,\ B(0) = 100,\ B(1) = 110;\ S(0) = 40 \text{ and}$$

$$S(1) = \begin{cases} 52, & \text{with probability } 0.4; \\ 36, & \text{with probability } 0.6. \end{cases}$$

   (a) Find the price, $C(0)$, of the call option.

   (b) Consider a portfolio $(100, 10, 50)$ of 100 shares, 10 bonds and 50 call options. Calculate the expected yield of the portfolio, and the associated risk.

3. Consider a market as in Question 2. Suppose now we also include put options, $P(t)$, with strike price $P_p = 40$.

   (a) Find the price of the put options, $P(0)$.

   (b) Calculate $E(i_P)$ and σ_{i_P}.

   (c) For the portfolio $(100, 10, 50, 50)$, consisting of 100 shares, 10 bonds, 50 call options and 50 put options find the expected yield and risk.

Question 4 over page

Question 4 will not be marked! Use it as a practice question.

4. An investor thinks there is a good chance that *GeneTherapy* stock will be go up dramatically over the next year because of a likely takeover bid, with the current price of $S(0) = 40$ dollars a share our investor estimates the share price in one years time as follows.

$$S(1) = \begin{cases} 80, & \text{with probability } 0.7; \\ 30, & \text{with probability } 0.3. \end{cases}$$

If the investor does not have the funds to invest in the shares today and the bond yield rate (or bank interest rate), i_B, is 10% compare the value of his possible portfolios at $t = 1$ (i.e. compare $V(1)$'s) if he borrows \$4,000 (at interest rate i_B) today and adopts one of the strategies listed below. In each case compare the possible gains and losses.

(a) Use the \$4,000 to buy shares of *GeneTherapy* today.
[Hint: First show that the value of the portfolio is $V(t) = -B(t) + \frac{4000}{40}S(t)$, with $B(0) = 4000$ and $B(1) = (1 + i_B)B(0)$.]

(b) Use the \$4,000 dollars to buy call options (price $C(0)$) on the stock with strike price \$40.
[Hint: First show that $V(t) = -B(t) + \frac{4000}{C(0)}C(t)$.]

(c) Use the \$4,000 to buy 300 put options (with strike price \$40), and then use the remainder of the money to buy call options.
[Hint: First show that $V(t) = -B(t) + 300P(t) + \frac{[4000-300P(0)]}{C(0)}C(t)$.]