MATH 2090.

Assignment 9.

Q1. Portfolio $(100, 40)$;
Portfolio Value :–

$$V(t) = 100\,S(t) + 40\,B(t)$$

(a) Initial Value $V(0) = 100\times 50 + 40\times 100$
$= 9{,}000$

i.e. $\$9{,}000$

Values at $t=1$:–

$$V(1) = \begin{cases} 100\times 80 + 40\times 110 & ; \text{prob. } 0.75 \\ 100\times 40 + 40\times 110 & ; \text{prob. } 0.25 \end{cases}$$

$$= \begin{cases} 12{,}400 & ; \text{prob. } 0.75 \\ 8{,}400 & ; \text{prob. } 0.25 \end{cases}$$

(b) Yields:

$$i_V = \frac{V(1) - V(0)}{V(0)}.$$

$$= \begin{cases} \dfrac{12{,}400 - 9{,}000}{9{,}000} & ; \text{prob. } 0.75 \\ \dfrac{8{,}400 - 9{,}000}{9{,}000} & ; \text{prob. } 0.25 \end{cases}$$

i.e. $$i_V = \begin{cases} 0.377' & ; \text{prob. } 0.75 \\ -0.066' & ; \text{prob. } 0.25 \end{cases}$$

(c) Question not clear return is the profit:

$$V(1) - V(0) = \begin{cases} 3{,}400 & ; \text{prob. } 0.75 \\ -600 & ; \text{prob. } 0.25 \end{cases}$$

Expected return is $E(V(1) - V(0)) = 3{,}400\times 0.75 - 600\times 0.25$
$= 2{,}400$

i.e. $\$2{,}400$

Expected yield is $E(i_V) = 0.377'\times 0.75 - 0.066'\times 0.25$
$= 0.266'$

(d) Risk is measured by the standard deviation of i_v

i.e. $$\sigma_{i_v} = \sqrt{\left\{(0.377' - 0.266')^2 \times 0.75 + (-0.066' - 0.266')^2 \times 0.25\right\}}$$

$$\doteqdot 0.192450$$

i.e. a 'risk' of 19.2450 %.

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2 (a)

Bob: $(150, 150)$: $V(t) = 150S(t) + 150B(t)$

so $V(0) = 150 \times 100 + 150 \times 100$
$= 30000$

i.e. initial cost is $\$30{,}000$.

Yield: $i_V = \dfrac{150S(1) + 150B(1) - 30000}{30000}$

$$= \begin{cases} \dfrac{215 \times 150 - 30000}{30000}, & \text{prob. } 0.75 \\ \dfrac{195 \times 150 - 30000}{30000}; & \text{prob } 0.25 \end{cases}$$

$$= \begin{cases} 0.075, & \text{probability } 0.75 \\ -0.025, & \text{probability } 0.25 \end{cases}$$

Expected Yield, $E(i_V) = 0.075 \times 0.75 - 0.025 \times 0.25$
$= 0.05$

Expected yield is 5%.

Sally: $(100, 200)$: $V(t) = 100S(t) + 200B(t)$
$= 30{,}000$

i.e. initial cost is $\$30{,}000$.

Yield $i_V = \begin{cases} \dfrac{200 \times 160 - 30000}{30000}, & \text{prob } 0.75 \\ \dfrac{100 \times 300 - 30000}{30000}, & \text{prob. } 0.25 \end{cases}$

$$= \begin{cases} \dfrac{1}{15}, & \text{prob } 0.75 \\ 0, & \text{prob } 0.25 \end{cases}$$

Expected yield, $E(i_V) = \frac{1}{15} \times 0.75 = 0.05$

Expected yield is 5%

So Bob's and Sally's portfolio's have the same initial cost and the same expected yield.

* For Bob's portfolio:

$$\sigma_{i_V} = \sqrt{\left\{(0.075-0.05)^2 \times 0.75 + (-0.025-0.05)^2 \times 0.25\right\}}$$

$$\doteq 0.0433013$$

So the risk for Bob's portfolio is $\doteq$ 4.33%.

* For Sally's portfolio:

$$\sigma_{i_V} = \sqrt{\left\{\left(\frac{1}{15}-0.05\right)^2 \times 0.75 + (0-0.05)^2 \times 0.25\right\}}$$

$$\doteq 0.0288675$$

So the risk for Sally's portfolio is $\doteq$ 2.89%.

So Sally's portfolio should be preferred to Bob's — the expected returns are the same but Sally's risk is quite a bit lower.

(c) For the portfolio $(0, 300)$ we have:—

$$V(t) = 300\,B(t)$$

$$V(0) = 300 \times 100 = 30\,000$$

i.e. initial cost is \$30,000, as before.

Yield: $$i_V = \frac{300\,B(1) - 30000}{30\,000}$$

$$= \frac{5}{100} = 0.05. \quad \text{as before.}$$

And as noted in lectures $\sigma_{i_v} = 0$ for the 'risk free bonds'.

3. We note that $Y = \alpha X + \beta = \begin{cases} \alpha x_1 + \beta, & \text{probability } p_1 \\ \alpha x_2 + \beta, & \text{probability } p_2 \end{cases}$

$(p_1 + p_2 = 1)$

Consequently,
$$E(Y) = (\alpha X_1 + \beta) \times p_1 + (\alpha X_2 + \beta) \times p_2$$
$$= \alpha (X_1 p_1 + X_2 p_2) + \beta (p_1 + p_2)$$
$$= \alpha E(X) + \beta$$
as $E(X) = X_1 p_1 + X_2 p_2$ and $p_1 + p_2 = 1$.

Now,
$$i_v = \frac{x S(0) i_S + y B(0) i_B}{x S(0) + y B(0)}$$

Note that on the r.h.s on i_S is a random variable: $\alpha = \dfrac{x S(0)}{x S(0) + y B(0)}$ and $\beta = \dfrac{y B(0) i_B}{x S(0) + y B(0)}$ are constants so $i_v = \alpha X + \beta$, where $X = i_S$

Apply previous formula with $Y = i_v$, so
$$E(i_v) = \alpha E(X) + \beta$$
$$= \alpha E(i_S) + \beta$$
$$= \frac{x S(0)}{x S(0) + y B(0)} E(i_S) + \frac{y B(0) i_B}{x S(0) + y B(0)}$$
$$= \frac{x S(0) E(i_S) + y B(0) i_B}{x S(0) + y B(0)}$$

4.

(a) $F = (1+i_B)S(0)$; $i_B = \frac{110-100}{100} = 0.1$, so

$F = 1.1 \times 50 = 55$. Contract price is \$55.

(b) $V(0) = 20 \times 50 + 100 \times 100$
$= 11,000.$

$V(1) = 20 \times S(1) + 100 \times 110 + 10 \times (S(1) - F)$

$= 30S(1) + 11000 - 55 \times 10$

$= 30S(1) + 10,450$.

yields:- $\therefore i_V = \frac{V(1)-V(0)}{V(0)} = \frac{30S(1) - 550}{11000}$.

$$= \begin{cases} 0.1, \text{ probability } 0.75, \\ 0.0754545, \text{ probability } 0.25. \end{cases}$$

Expected yield:

$E(i_V) = 0.1 \times 0.75 + 0.0754545 \times 0.25$
$= 0.0938636$

i.e. expected yield is (approximately) 9.39%.

(c) Standard deviation for yield:-

$$\sigma_{i_V} = \sqrt{\left\{[0.1 - 0.0938636]^2 \times 0.75 + [0.0754545 - 0.0938636]^2 \times 0.25\right\}}$$

$\doteqdot 0.0106254$

i.e. risk of about 1.06%.