Q1

MATH2090 Assignment 8.

Coupon is given as semi-annual so $r = \frac{1.8}{100} = 0.018$

[Note: coupon is not given as 1.8% convertible semi-annually, it is semi-annual]

(a) (i) 2.0% semi-annual yield so $i = \frac{2.0}{100} = 0.02$

Note $i > r$ so bond is at discount to face value. This means that

- Book-Value increases from $t = 0$ to $t = n$.
- The Price for a callable bond decreases from $n = 17 \times 2 = 34$, to, $n = 20 \times 2 = 40$.

If bond is redeemed at the begining of the 18th year then $n = 17 \times 2$ and Price = \$951.00(282). If redeemed at end of 20th year Price = \$945.28(904).

So to get the 2% semi annual yield the investor needs to pay at least \$945.28 but no more than \$951.00.

(ii) For $i = \frac{1.4}{100} = 0.014$ we have $r > i$ so the bond is at a premium. In this case the price increases with the time:-

$n = 34$, Price = \$1,107.82(35)

$n = 40$, Price = \$1,121.87(66)

[Note: Suppose $N_2 > N_1$ and P_N is price of callable bond then $P_{N_2} = \frac{Fr}{i}(1 - v^{N_2}) + Fv^{N_2}$

$$P_{N_1} = \frac{Fr}{i}(1 - v^{N_1}) + Fv^{N_1}$$

$$P_{N_2} - P_{N_1} = F(v^{N_1} - v^{N_2})\left(\frac{r}{i} - 1\right)$$

Now $v < 1$ so $v^{N_1} - v^{N_2} > 0$

So for $\frac{r}{i} > 1$ $P_{N_2} > P_{N_1}$ and for $\frac{r}{i} < 1$, $P_{N_2} < P_{N_1}$.]

(b) At the end of the 18^{th} year $n = 2 \times 18 = 36$.

From lectures

$$\bar{d} = \frac{vFr}{1-v} \cdot \frac{\left[\left(\frac{1-v^n}{1-v}\right) - n\right]}{\left[Frv\left(\frac{1-v^n}{1-v}\right) + E v^n\right]} + n$$

$F = C = 1000$, $n = 36$, $r = 0.018$

(i) $i = 0.02$, $v = \frac{1}{1.02}$ so

$\bar{d} \doteq 26.515247$, $\bar{v} = \frac{\bar{d}}{1+i} = 25.99534$

(ii) $i = 0.014$, $v = \frac{1}{1.014}$ so

$\bar{d} \doteq 27.356107$, $\bar{v} = \frac{\bar{d}}{1+i} = 26.978409$

2. (a) $r = \frac{3.8}{4 \times 100} = 0.0095$, $F = C = 1000$ and

$n = 4 \times 10 = 40$

Quarterly yield i :- $(1+i)^4 = 1 + \frac{2.4}{100} = 1.024$

$\therefore \; i = (1.024)^{\frac{1}{4}} - 1$

$\doteqdot 0.0059467$

$$\text{Price, } P = 1000 \times 0.095 \times \frac{[1 - (1.024)^{-40/4}]}{(1.024)^{\frac{1}{4}} - 1} + 1000 \times (1.024)^{-\frac{40}{4}}$$

$$\doteqdot 1126.1583$$

i.e. standard coupon bond is priced at $\underline{\$1,126.15(83)}$.

(b) This particular split (or strip) bond pays everything at maturity:

$$\text{Price, } P = (1000 + 40 \times 0.0095 \times 1000) \times (1.024)^{-\frac{40}{4}}$$

$$\doteqdot 1088.628$$

i.e. price of the split bond is $\underline{\$1,088.62(8)}$.

3. Portfolio $(60, 30)$ has value

$$V(t) = 60\, S(t) + 30\, B(t)$$

(a) At $t = 0$: $V(0) = 60 \times 80 + 30 \times 100$
$= 7800$

At $t = 1$: $V(1) = 60\, S(1) + 30 \times 110$

$$= \begin{cases} 9,300; & \text{probability } 0.8 \\ 6,900; & \text{probability } 0.2. \end{cases}$$

(b) Now $i_V = \frac{V(1) - V(0)}{V(0)}$

So, $i_V = \begin{cases} \frac{5}{26}; & \text{with probability } 0.8 \\ -\frac{3}{26}; & \text{probability } 0.2 \end{cases}$