F17

MATH2090 – Mathematics of Finance

Assignment 7

Name:

MUN Number:

Due Date: Monday, 6 November

1. A bond of \$10,000 redeemable at par after 10 years has a coupon of 2.9% per year convertible semi-annually. Find the price which will yield the investor

   (a) 2% effective per half year. (b) 4% effective per year.

2. A par-value bond with a \$10,000 face value and 2.6% coupon, convertible semi-annually, is priced at \$11,000. If the effective annual yield rate is to be no more than 1.8% what is the maximum whole year term (i.e. time to maturity) of the bond?

3. A corporation issues par-value bonds with an annual 2.9% coupon yielding 2.5% effective annually and with a ten year maturity. The corporation now proposes to issue new par-value bonds with the same price and face value but with a coupon rate of 3.2%, convertible semi-annually. What is the maximum whole year term that this the new bond must have in order that they give an annual yield which is no more than the original bonds?

4. Consider a par-value bond with a 2.4% coupon, convertible quarterly (i.e. every 3 months), face value of \$1,000 with a ten year maturity and an effective annual yield rate of 2.9%.

   (a) What is the price of this bond?

   (b) If the bond is bought at the beginning of January, 2014, find the book value of the bond on June 15, 2020.

   (c) Give the bond amortization schedule for the year 2023, assuming it is bought at the beginning of 2014.

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ASSIGNMENT 7 – Solutions

Q.1. Our formula is $P = Fr\, a_{\overline{n}|i} + C(1+i)^{-n}$
where i is the yield rate.

$$r = \frac{2.9}{2 \times 100} = 0.0145, \quad F = C = 10\,000$$

(a) $i = \frac{2}{100} = 0.02$

$$\therefore P = 145 \times \frac{\left[1-(1.02)^{-2\times 10}\right]}{0.02} + 10000\,(1.02)^{-2\times 10}$$

$$\doteq 9100.6712$$

The bond price is \$9,100.67(12)

(b) yield, i, determined by

$$(1+i)^2 = 1 + \frac{4}{100}$$

i.e. $1+i = \left(1+\frac{4}{100}\right)^{\frac{1}{2}} \doteq 1.0198039$

to give the yield per 6 months.

then the price is

$$P = 145\,\frac{\left[1-(1.04)^{-\left(\frac{2\times 10}{2}\right)}\right]}{(1.04)^{\frac{1}{2}}-1} + (1.04)^{-\left(\frac{2\times 10}{2}\right)}$$

$$\doteq 9131.0924$$

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2. Use the price equation with
$P = 11{,}000$, $C = F = 10{,}000$, $r = \frac{2.6}{2\times 100} = 0.013$

and semi annual yield i satisfying

$$(1+i)^2 \leq 1.018 \text{, i.e. } i \leq \sqrt{1.018} - 1$$

Price equation gives

$$11\,000 = 10\,000 \left\{ 0.013 \frac{[1-(1+i)^{-n}]}{i} + (1+i)^{-n} \right\}$$

i.e. $$1.1 = 0.013 \frac{[1-(1+i)^{-n}]}{i} + (1+i)^{-n}$$

Put $y = 1+i$ $(i = y-1)$:

$$1.1 = 0.013 \frac{[1-y^{-n}]}{y-1} + y^{-n}$$

i.e. $$y^n [1.1 \times (y-1) - 0.013] = y - 1.013$$

i.e. $$y^n = \frac{y - 1.013}{1.1 \times y - 1.113}$$

$$\therefore \quad n = \frac{\ln \left[\frac{y-1.013}{1.1\times y - 1.113} \right]}{\ln y}$$

with $y = 1+i = \sqrt{1.018}$ we get

$$n \doteqdot 28.109382$$

Now do a little experimenting to check if n goes up or down with (e.g. if $y = 1+i = \sqrt{1.019}$ then $n \doteqdot 32.96\ldots$)

* So the time to maturity must be $\frac{1}{2} \times 28 = \underline{14 \text{ years}}$.

Q.3. Let F be the face value of both bonds.

For first bond :–

- annual coupon, $r = 0.029$.
- annual yield, $i = 0.025$.
- $n = 10$.

$$\therefore \quad P_1 = F\{0.029\, a_{\overline{10}|i} + (1\cdot 0.025)^{-10}\}$$

$$\doteqdot F \times 1.03500825572388$$

For the second bond :–

- Semi-annual coupon, $r = 0.016$
- Semi annual yield, j, with

$$(1+j)^2 \leqslant 1+0.025 = 1.025,$$

$$\text{ie. } j \leq 0.01242284.$$

- $n = 2N$ where N is the time to maturity (in years).

$$\therefore \quad P_2 \doteqdot F\left\{0.016\left[\frac{1-(1+j)^{-2N}}{j}\right] + (1+j)^{-2N}\right\}$$

Require $P_1 = P_2$ ~~so~~

$$(*) \quad 1.03500825572388 \doteqdot \frac{0.016}{j} - \left(\frac{0.016}{j} - 1\right)(1+j)^{-2N}$$

and as, $j \leq 0.01242284 < 0.016$

we note $\dfrac{0.016}{j} - 1 > 0$

With $j = 0.01242284$ (ie. annual yield 2.5%) we get from $(*)$:

$$-2N = \frac{\ln\left\{\dfrac{\dfrac{0.016}{j} - 1.03500825572388}{\left[\dfrac{0.016}{j} - 1\right]}\right\}}{\ln(1+j)}$$

$$= -10.4992823459028$$

$$\Rightarrow \quad N \doteqdot 5.24964173$$

So we get bond maturity date of 5 years.

Now check the yield with $N = 5$:-

From $(*)$:

$$1.0350082\cdots = \frac{0.16}{j} - \left(\frac{0.016}{j} - 1\right)(1+j)^{-10}$$

Put $x = 1+j$, $j = x-1$:

$$1.0350082\cdots = \frac{0.16}{x-1} - \left(\frac{0.016}{x-1} - 1\right)x^{-10}$$

i.e.

$$(1.0350082\cdots)(x-1)x^{10} = 0.016\,x^{10} - \left[0.016 - (x-1)\right]$$

$$\therefore \quad (1.0350082\cdots)x^{11} - \left[(1.0350082\cdots) + 0.016\right]x^{10} - x + 1.016 = 0$$

MATLAB Roots gives

$$x \doteq 1.012258823554 45$$

$$\doteq 1+j$$

the annual yield is $(1+j)^2 - 1 \doteq 0.024668$

i.e. an annual yield of $2.4668\,\%$.

4. Coupon is convertible quarterly, ie. 4 times per year so $r = \frac{2\cdot4}{4\times100} = 0\cdot006$, the quarterly yield rate i is given by

$$(1+i)^4 = 1+\frac{2\cdot9}{100}, \text{ ie. } 1+i = (1\cdot029)^{\frac{1}{4}}$$

a) Price of bond,

$$P = 1000\times0\cdot006\,\frac{\left[1-(1\cdot029)^{-\frac{40}{4}}\right]}{(1\cdot029)^{\frac{1}{4}}-1} + 1000\,(1\cdot029)^{-\frac{40}{4}}$$

$$\doteqdot 959\cdot35495.$$

ie. the bond price is $\$959.35(495)$

(b) Jun 15, 2020 is $6\times4+1+\frac{2\cdot5}{3} = 25\frac{5}{6}$ quarters from January 1, 2014. So we want the book value for $k = 25\frac{5}{6} = 25\cdot8333'$

$$B_k = 1000\times0\cdot006\,\frac{\left[1-(1\cdot029)^{-\frac{(40-25\cdot8333)}{4}}\right]}{(1\cdot029)^{\frac{1}{4}}-1} + 1000\times(1\cdot029)^{-\frac{(40-25\cdot8333)}{4}}$$

$$\doteqdot 984\cdot25965$$

ie. the required book value is $\$984.25(965)$

(c) First we need the book value at the end of 2022 (ie. beginning of 2023), B_k, with $k = 4\times8 = 32$ quarters

$$B_{32} = 1000\times0\cdot006\,\frac{\left[1-(1\cdot029)^{-\frac{(40-32)}{4}}\right]}{(1\cdot029)^{\frac{1}{4}}-1} + 1000\times(1\cdot029)^{-\frac{(40-32)}{4}}$$

$$\doteqdot 990\cdot91593$$

Time	Coupon	Interest	Principle Adjustment	Book Value
End 2022	—	—	—	990.91593
1st quarter	6	7.1073088	1.1073088	992.02324
2nd quarter	6	7.115251	1.115251	993.13849
3rd quarter	6	7.1232501	1.1232501	994.26174
End 2023	6	7.1313065	6.1313065	995.39305

Bond Amortization Schedule 2023