(1)

MATH 2090.

Assignment 6 Solutions.

1. First, the interest rate is annual 5%, we need the effective 6 monthly rate j:–

$$(1+j)^2 = 1 + \frac{5}{100} = 1.05$$

$$\therefore\ j = \sqrt{(1.05)} - 1 \doteqdot 0.024695$$

Next, the loan repayments X:

$$10000 = X\, a_{\overline{2\times 4}|j} = X\, a_{\overline{8}|j}$$

$$\text{So } X = \frac{10000}{a_{\overline{8}|j}}$$

i.e.

$$X \doteqdot 1392.860306$$

i.e. $1,392.86(0306).

Duration	Payment	Interest Portion	Principal Portion	Outstanding Principal
0				10,000
1	1392.56(03)	246.95(08)	1145.90(95)	8854.09(05,
2	1392.86(03)	218.65(24)	1174.20(79)	7679.88(26,
3	1392.86(03)	189.65(53)	1203.20(50)	6476.67(76
4	1392.86(03)	159.94(20)	1232.91(83)	5243.75(9
5	1392.86(03)	129.49(50)	1263.36(53)	3980.39(40
6	1392.86(03)	98.29(61)	1294.56(42)	2685.82(98
7	1392.56(03)	66.32(68)	1326.53(35)	1359.29(23
8	1392.56(03)	33.56(79)	1359.29(23)	0

2. (a) Annual Interest Payment $= 15000 \times \frac{5.3}{100}$
$= 795$

i.e. $\underline{\$795.00}$.

(b) Sinking fund payments are monthly, let monthly rate be j, then

$$j = (1.042)^{\frac{1}{12}} - 1 \doteqdot 0.003434$$

Equation of accumulated value gives

$$15000 = X\, s_{\overline{5\times 12}|j} \quad , X \text{ the monthly payment.}$$

So, $x = \dfrac{15000}{s_{\overline{60}|i}} \doteq 225.553692$

i.e. monthly sinking fund payment is $\$225.55$ (Near Cent)

(c) Annual outlay = annual interest + 12 monthly payments

$= 795 + 12 \times 225.553692$

$= 3501.644301$

i.e. $\$3,501.64$ (Nearest Cent).

First, consider ~~monthly~~ payment Y on loan with 6.3% annual interest

$$Y = \frac{15000}{a_{\overline{60}|j_1}}, \quad j_1 = (1.063)^{\frac{1}{12}} - 1$$

i.e. $Y \doteq 290.865295$

Annual outlay for this amortized loan is $12 \times Y$

i.e. $\$3490.38(3546)$.

So for the sinking fund loan we require 12 payments with value

$$3490.38(3546) - 795 = 2695.383546$$

i.e. monthly sinking fund payments of

$$2695.38546 \div 12 = 224.615295$$

Let the monthly sinking fund interest rate be j_2

so $15000 = 224.615295 \, s_{\overline{60}|j_2}$

OR $s_{\overline{60}|j_2} = 66.780848$

i.e. $$\frac{(1+j_2)^{60} - 1}{j_2} = 66.780848$$

Put $z = 1 + j_2$ to get (with $j_2 = z - 1$)

)

$$\frac{z^{60}-1}{z-1} = 66.780848$$

i.e. $z^{60} - 66.780848\,z + 65.780848 = 0.$

Using Matlab roots function, we find there is only one root $z > 1$, it is $z = 1.00357133$

$\therefore$ $j_2 = 0.00357133$ is the monthly interest rate.

The corresponding annual rate i_2 is given by

$$1 + i_2 = (1 + j_2)^{12} \doteq 1.043708$$

i.e. the required annual rate is 4.3708%.

3. Firstly, the monthly rate j_1 for the initial loan;

$$j_1 = (1.074)^{\frac{1}{12}} - 1 \doteq 0.00596690$$

the payments on this initial loan are

$$\frac{10\,000}{a_{\overline{5\times 12|}\,j_1}} = 198.769050$$

~~The outstanding principal just after the 12th payment is (prospective method)~~

~~198.769050~~

Now Vulture will receive 4 years of monthly payments of \$198.76(9050) at an interest rate of 14% annualy, which is $j_2 = (1.14)^{\frac{1}{12}} - 1$ monthly

So Vulture pays, using the present value equation

$$198.76(9050) \times a_{\overline{4\times 12|}\,j_2} = \underline{7385.272702}$$

i.e. \$ 7,385.27 (Nearest Cent)

[Note: This is of course much less than the outstanding

1 principal on original loan, which is \$8,274·91)

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4. The initial loan i—
Monthly rate j is given by $1+j = \left(1+\frac{5\cdot 1}{2\times 100}\right)^{\frac{1}{6}}$
$\doteq 1\cdot 00420553$

Repayments are: $\frac{145{,}000}{a_{\overline{20\times 12}|j}} \doteq 960\cdot 675836$.

At the end of 10 years the outstanding principal can be calculated using the prospective method

Outstanding principal after 10 yrs $= 960\cdot 675836 \times a_{\overline{10\times 12}|j}$

$= 90379\cdot 424045$.

(a) In this case Hilary (after paying \$25,000)
owes \$65,379·42(4045).
The interest rate is now 5·4 % (convertible semi-annually)
to give monthly j_2: $1+j_2 = \left(1+\frac{5\cdot 4}{2\times 100}\right)^{\frac{1}{6}}$
$\doteq 1\cdot 00445019$

So her repayments for remaining 10 years are

$\frac{65{,}379\cdot 424045}{a_{\overline{10\times 12}|j_2}} \doteq 704\cdot 373960$

i.e. \$704·37(3960).

— Yield rate are calculated later.

(b) Annual inflation is 2·1 % monthly this is
$1+r = \left(1+\frac{2\cdot 1}{100}\right)^{\frac{1}{12}} = 1\cdot 00173338$
there are 120 payments:
$P,\ P(1+r),\ P(1+r)^2,\ P(1+r)^3, \dots, P(1+r)^{119}$
From Assignment 5 the present value of these payments

is $P.V = P\nu + P(1+r)\nu^2 + \cdots + P(1+r)^{117}\nu^{120}$

$$= \nu P\left[1 + (1+r)\nu + (1+r)^2\nu^2 + \cdots + (1+r)^{119}\nu^{119}\right]$$

$$= \nu P\left[\frac{1-(1+r)^{120}\nu^{120}}{1-(1+r)\nu}\right]$$

where $\nu = \dfrac{1}{1+j} = \dfrac{1}{1\cdot00420553}$, r as above.

So $P.V. = 103\cdot573293 \times P$, P first repayment as above

$= $ outstanding principal $65,379\cdot42(4045)$

$$\therefore \quad P = \frac{65,379\cdot424045}{103\cdot573293}$$

$= 631\cdot238249$, or $\$631\cdot23(8249)$

So the 120 payments are

$631\cdot238249, 632\cdot332424, \ldots, 775\cdot708557$

or $631\cdot23(8249) \times (1\cdot00173338)^{k-1}$, $k = 1, 2, \ldots, 120$

— payments increase with inflation.

(c) In this case $65,379\cdot42(4045) = X\, a_{\overline{12\times5}|j}$

j the monthly rate in (a).

So $X = 1235\cdot187950$

i.e. repayments are $\$1,235\cdot18(7950)$

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Yields questions a little unclear how these should be calculated. Method was meant to be simplistic $145000(1+i)^n = $ Sum all payments.

$n = 20$ for (a) and (b) or $n = 15$ for (c) for (a) :-

$$145000(1+i)^{20} = 120 \times 960.675836 + 120 \times 704.373960 + 25{,}000$$

i.e. $$1+i = \left[\frac{224805.97552}{145000}\right]^{\frac{1}{20}} = 1.022167$$

i.e. a rate of $\underline{2.2167\%}$.

A more sophisticated way is to apply the rate i to all payments to the lender. To do this need monthly rate $j = (1+i)^{\frac{1}{12}} - 1$:—

$$145000(1+j)^{12\times 20} = 960.675836\, s_{\overline{120}|j} + 704.373960\, s_{\overline{120}|j} + 25000(1+j)^{120}$$

With $x = 1+j$ this gives

$$x^{240} = 0.006625\frac{[x^{120}-1]}{x-1} + 0.004858\frac{[x^{120}-1]}{x-1} + 0.172414\, x^{120}$$

i.e. $$(x-1)x^{240} - 0.011483(x^{120}-1) - 0.172414\, x^{120}(x-1) = 0$$

i.e. $$x^{241} - x^{240} - 0.172414\, x^{121} + 0.160931\, x^{120} + 0.011483 = 0$$

MATLAB roots gives just one root >1, so

$$x \doteq 1.00255328, \text{ as } x = 1+j$$

$\therefore$ $j = 0.00255328$ is monthly rate

the annual rate $i = (1+j)^{12} - 1$

$$\doteq 0.031073$$

i.e. a rate of $\underline{3.1073\%}$.