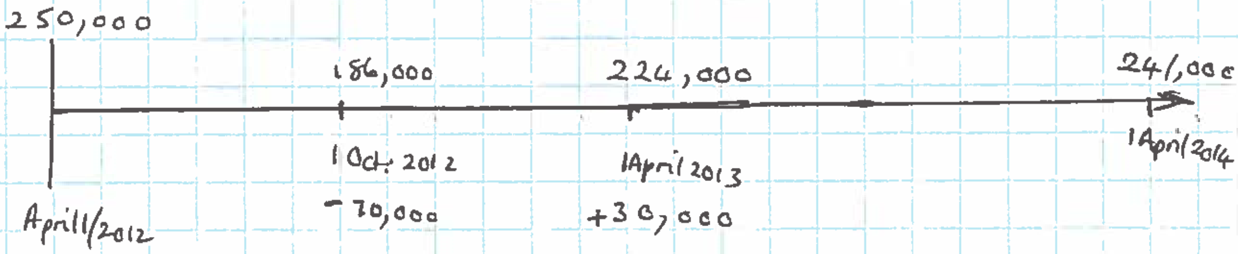
Assignment 3

1. Timeline:-



(a)

* Need to take ~~care~~ because when we calculate a time-weighted return we need the balance immediately before each deposit / withdrawl.

In this problem the balances are after the deposit/withdrawls!

The effective interests rates i_1 (1 April 2012 to 1 Oct. 2012), i_2 (1 Oct 2012 to 1 April 2013) and i_3 (1 April 2013 to 1 April 2014) are given by:- B_j = balance just before deposit W_j.

— $1+i_1 = \dfrac{B_1}{B_0}$, $B_0 = 250{,}000$

$B_1 = 186{,}000 + 70{,}000 = 256{,}000$

$\therefore\ 1+i_1 = \dfrac{256{,}000}{250{,}000} = \dfrac{256}{250} = \dfrac{128}{125}$

— $1+i_2 = \dfrac{B_2}{B_1 + W_1}$, $B_2 = 224{,}000 - 30{,}000 = 194{,}000$

$W_1 = -70{,}000$

$= \dfrac{194{,}000}{186{,}000} = \dfrac{97}{93}$

— $1+i_3 = \dfrac{B_3}{B_2 + W_2}$, $B_3 = 241{,}000$

$W_2 = +30{,}000$

$= \dfrac{241{,}000}{224{,}000} = \dfrac{241}{224}$

The time-weighted rate of return for the whole two years is

$$i_t = (1+i_1)(1+i_2)(1+i_3) - 1$$

$$= \frac{128}{125} \cdot \frac{97}{93} \cdot \frac{241}{224} - 1$$

$$\doteq 0.1490998$$

i.e. approximately (2 places) 14.91 %. //

(b) To calculate the dollar-weighted rate of return, i, we need the equation of value:–

$$241{,}000 = 250{,}000 \times (1+i)^2 - 70{,}000 (1+i)^{\frac{6}{12}+1} + 30{,}000 (1+i)^1$$

i.e. $$25 \times (1+i)^2 - 7 \times (1+i)^{\frac{3}{2}} + 3 \times (1+i) - 24.1 = 0$$

Write $x = (1+i)^{\frac{1}{2}}$ so we have

$$25x^4 - 7x^3 + 3x^2 - 24.1 = 0$$

Use the 'roots' function on SCILAB (MATLAB)

roots([25 −7 3 0 −24.1]) gives

-0.9017215
$0.0735762 \pm 1.0138693i$ ← $\sqrt{-1}$
1.0345692

Now $x = \sqrt{1+i} > 1$ so we have

$$x \doteq 1.0345692$$ so

$$i = x^2 - 1 \doteq 0.0703334$$

i.e. approximately (2 places) 7.03% //

Q2.

(a) Use Geometric Series formulae: $a_n = ar^{n-1}$, $S_n = a\left(\frac{1-r^n}{1-r}\right)$

For given sequence $a = 9$, $r = -\frac{1}{3}$ (ratio of terms)

$\therefore$ n^{th} term is $9 \times \left(-\frac{1}{3}\right)^{n-1} = (-1)^{n-1}\frac{9}{3^{n-1}} = \frac{(-1)^{n-1}}{3^{n-3}}$

Sum of first n terms, $S_n = 9\left[\frac{1-\left(\frac{-1}{3}\right)^n}{1-\left(-\frac{1}{3}\right)}\right]$

$$= \frac{9}{1+\frac{1}{3}}\left[1-(-1)^n\frac{1}{3^n}\right]$$

i.e. $S_n = \frac{27}{4}\cdot\left[1-(-1)^n\frac{1}{3^n}\right]$

(b) Definition:- $a_{\overline{n|}} = \frac{1-v^n}{i}$.

Now $a_{\overline{m+n|}} = \frac{1-v^{m+n}}{i} = \frac{v^m(1-v^n) - v^m + 1}{i}$

$$= \left(\frac{1-v^n}{i}\right)v^m + \frac{1-v^m}{i}$$

i.e. $a_{\overline{m+n|}} = v^m a_{\overline{n|}} + a_{\overline{m|}}$, as required

(c) Definition:- $s_{\overline{n|}} = \frac{(1+i)^n - 1}{i}$,

Note $a_{\overline{n|}} = \frac{1-v^n}{i}$ and $s_{\overline{n|}} = \frac{v^{-n}-1}{i} = \frac{1-v^n}{i\,v^n}$

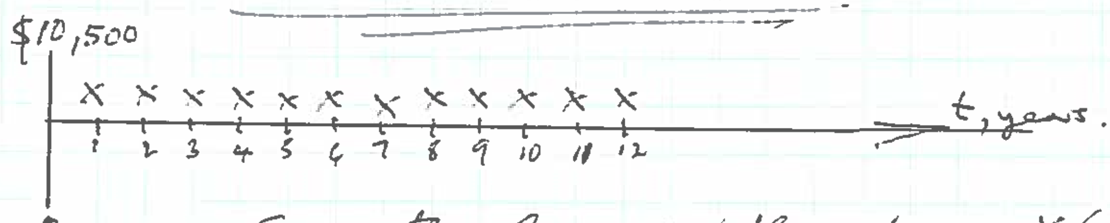
(as $(1+i)^n = v^{-n}$), hence

$$\frac{1}{a_{\overline{n|}}} - \frac{1}{s_{\overline{n|}}} = \frac{i}{1-v^n} - \frac{i v^n}{1-v^n}$$

$$= \frac{i}{1-v^n}\cdot\left(1 - v^n\right) = i$$

i.e. $\frac{1}{a_{\overline{n|}}} - \frac{1}{s_{\overline{n|}}} = i$, as required.

Q.3.



Suppose the loan installments are X (dollars) as shown in diagram.

Now $i = \frac{4.5}{100} = 0.045$, so $v = \frac{1}{1.045}$

So from the equation of Present Value we have

$$10500 = X\, a_{\overline{12|}} \quad \text{and} \quad a_{\overline{12|}} = \frac{1-v^{12}}{i}$$

$$= \frac{1 - \frac{1}{(1\cdot045)^{12}}}{0\cdot045}$$

$$\doteq 9\cdot0431101$$

So, $X \doteq \frac{10500}{9\cdot0431101} = 1161\cdot105$

i.e. repayments are \$1161·11 (nearest cent)

Q4. Need to find the monthly interest rate, j say, which coincides with monthly repayments.

Now $(1+j)^{12} = 1+i = 1\cdot045$, or $1+j = (1\cdot045)^{\frac{1}{12}}$

$$\therefore \quad 1+j \doteq 1\cdot0036748$$

$$\text{i.e. } j \doteq 0\cdot0036748$$

Let X be the monthly repayments then the present value equation is

$$10500 = X\, a_{\overline{12\times12|}j}$$, Use $a_{\overline{12\times12|}j}$ to emphasise interest rate j

$$a_{\overline{12\times12|}j} = \frac{1 - \frac{1}{(1+j)^{144}}}{j} = \frac{1 - \frac{1}{(1\cdot045)^{12}}}{0\cdot0036748}$$

$$\doteq 111\cdot66217$$

$$\therefore \quad X = \frac{10500}{a_{\overline{144|}j}} \doteq 94\cdot033637$$

i.e. monthly repayments are \$94·03 (Nearest Cent)

[Note this equates to $12 \times 94\cdot033637 = 1128\cdot4036$ dollars per year slightly less than Q3.]

Q5. (a) This is just the present value of the annuity $i = 2\cdot17/100$, $v = \frac{1}{1\cdot0217}$

So $a_{\overline{25|}} = \frac{1-v^{25}}{i} = \frac{1-\frac{1}{(1.0217)^{25}}}{0.0217}$

So the required present Value is

$$45000 \times a_{\overline{25|}} = 45000 \times \left[\frac{1-\frac{1}{(1.0217)^{25}}}{0.0217}\right]$$

$$\doteqdot 861,271.25.$$

i.e. $\$861,271.25$

(b) The value of the annuity at the last payment is $45,000 \times s_{\overline{25|}}$, but we require its value one year later — one more compounding!

Answer is $45,000 \times s_{\overline{25|}} \times (1+i)$

$$= 45000 . \left[\frac{(1+i)^{25}-1}{i}\right](1+i)$$

$$= 45000 . \frac{\left[(1.0217)^{25}-1\right](1.0217)}{0.0217}$$

$$\doteqdot 1505040.5$$

i.e. Value one year after last payment is $\$1,505,040.50$

(c) Value at end of year ten

$=$

{Accumulated Value of payment at year 10} +
{Present Value (at year 10) of the contributions years 11 to 25}

$=$

$$45000\, s_{\overline{10|}} + 45000\, a_{\overline{25-10|}}$$

$$= 45000\left[\frac{(1.0217)^{10}-1}{0.0217} + \frac{1-\frac{1}{(1.0217)^{15}}}{0.0217}\right] \doteqdot 1067514.80$$

i.e. $\$1,067,514.80$.

(d) The present value for 25 years is (see (a) above) \$861,271.25. If we have n annual payments of \$45,000 so that the present value is $2 \times 861,271.25$ then the present value equation is

$$2 \times 861,271.25 = 45000 \times a_{\overline{n|}}$$

Need to find n :—

$$a_{\overline{n|}} = \frac{1-(1.0217)^{-n}}{0.0217}$$

so

$$2 \times 861,271.25 = 45000 \times \left[\frac{1-(1.0217)^{-n}}{0.0217}\right]$$

and so, with a little algebra (!),

$$1-(1.0217)^{-n} = \frac{0.0217}{45000} \times 2 \times 861271.25$$

i.e. $$(1.0217)^{-n} = 1 - \frac{0.0217 \times 2 \times 861271.25}{45000}$$

Take ln both sides:

$$-n \ln(1.0217) = \ln\left[1 - \frac{0.0217 \times 2 \times 861271.25}{45000}\right]$$

$$\therefore n = \frac{-\ln\left[1 - \frac{0.0217 \times 2 \times 861271.25}{45000}\right]}{\ln(1.0217)}$$

$$\doteqdot 82.717776$$

To the nearest day this is 82 years and 262 days to double the current present value.