MATH 2090 — Math of Finance

Assignment 1, F17.

1\. (a) Simple Interest : $a(t) = 1 + it$

$A(t) = P(1+it)$

For $i = \frac{3.1}{100} = 0.031$, $P = 3{,}300$ and time

$t = 2017 - 2002 = 15$ years we have

$A(15) = 3300.(1 + 0.031 \times 15) = 4834.5$

i.e. amount accumulated is $\$4{,}834.50$

(b) Compound Interest : $a(t) = (1+i)^t$

$A(t) = P(1+i)^t$

P, i and t are same as (a), above, so

$A(15) = 3300.(1+0.031)^{15} \doteqdot 5216.676660$

i.e., to nearest cent, amount accumulated is $\$5{,}216.68$

(c) Need the number of days from April 1, 2002 to September 10, 2017: Can count them on a calendar (takes time!) or just Google: 5641 days.

So $t = \frac{5641}{365}$ for the time to be used in exact simple interest, i.e. $t = 15.454795$

and $A\left(\frac{5641}{365}\right) = 3300.\left(1 + 0.031 \times \frac{5641}{365}\right)$

$\doteqdot 4881.025479$

i.e., to nearest cent, amount accumulated is $\$4{,}881.03$

(d) Use compound interest with t as in (c),

i.e. $A\left(\frac{5641}{365}\right) = 3300(1+0.031)^{\frac{5641}{365}}$

$\doteqdot 5289.612856$

i.e., to nearest cent, the accumulated amount is $\$5{,}289.61$

2. (a) 7·9% is the nominal annual rate, its converted monthly so there are 12 compoundings in the year. The effective annual rate, i, is given by

$$1+i = \left(1 + \frac{7\cdot9}{12 \times 100}\right)^{12}$$

$$= 1\cdot0819241698$$

i.e. ~~an~~ effective annual rate of 8·19% (Two dec. pl.)

(b) Dodgey's effective annual rate is 8·19% Whereas Real's annual rate is 8% so Dodgey offers the best investment.

Accumulated value of Dodgey investment is

$$A_D = 1200\left(1+ \frac{7\cdot9}{12\times100}\right)^{12\times5}$$

$$\doteqdot 17789\cdot566283$$

Real's accumulated value is

$$A_R = 1200\cdot\left(1+\frac{8}{100}\right)^{5}$$

$$\doteqdot 17631\cdot936922$$

$$A_D - A_R \doteqdot 157\cdot629332 \text{ (6 dec. places)}$$

i.e. (Nearest cent) cash difference is $157·63

Q. 3. (a) Interest rate is 2·7% (nominal) compounded monthly.

Twelve payments of \$4,250 at the beginning of each month. Interest is calculated at the end of each month.

Accumulated value at end of first year:-

$$A(1) = 4250\left(1+\frac{2.7}{12\times100}\right)^{12\times1} + 4250\left(1+\frac{2.7}{12\times100}\right)^{11} + \cdots + 4250\left(1+\frac{2.7}{12\times100}\right)^{1}$$

first payment compounds 12 months — second payment compounds 11 months — 12th payment compound 1 month.

$$= 4250\left[r^1 + r^2 + \cdots + r^{12}\right], \text{ where } r = 1+\frac{2.7}{12\times100}$$

[use series sum or direct calculation.]

$$\doteq 51752.063223$$

i.e. the accumulated value is (nearest cent) \$51,752·06

(b) Looks complicated at first! But remember these costs can be thought of as negative payments into the account. So these costs contribute:

$$-2350\times\left(1+\frac{2.7}{12\times100}\right)^{9} - 2350\times\left(1+\frac{2.7}{12\times100}\right)^{6} - 2350\times\left(1+\frac{2.7}{12\times100}\right)^{3} - 2350\times\left(1+\frac{2.7}{12\times100}\right)^{0}$$

$$= -2350\left[1 + r^3 + r^6 + r^9\right], \quad r \text{ as in (a)}.$$

$$\doteq -9495.820250$$

Combining this with (a) the accumulated value is 42256·242972

i.e. \$42,256·24 (nearest cent)

3. ctd (c). The Tax man is getting 40% of (b) that leaves 60% as the final accumulated value — just the after tax profit of the investment

this is $\frac{60}{100} \times 42256.262972$

$\doteq 25353.745783$

i.e. $\underline{\$25,353.75}$ (nearest cent).

4. (a)

$$i_n = \frac{a(n) - a(n-1)}{a(n-1)}$$

$$= \frac{\left(\frac{2}{1+e^{-n}}\right) - \left(\frac{2}{1+e^{-(n-1)}}\right)}{\left(\frac{2}{1+e^{-(n-1)}}\right)}$$

$$= \frac{1+e^{-(n-1)}}{1+e^{-n}} - 1$$

i.e. $$i_n = \frac{e^{-(n-1)} - e^{-n}}{1+e^{-n}}$$

as $n \rightarrow$ $e^{-(n-1)}$, $e^{-n} \longrightarrow 0$

and $1+e^{-n} \longrightarrow 1+0$

$\therefore$ $i_n \longrightarrow \frac{0-0}{1+0} = 0$ as $n \longrightarrow \infty$.

(b) Force of interest $\delta(t) = \frac{a'(t)}{a(t)} = \frac{1}{a(t)} \frac{da}{dt}$

$$= \frac{-2(1+e^{-t})^{-1-1} \times (-e^{-t})}{\left(\frac{2}{1+e^{-t}}\right)}$$

$$= \frac{e^{-t}}{1+e^{-t}}$$