MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 1000 **QUIZ #2** – *Nov. 16, 2009* **NAME:**

Marks

15 1. Find the derivative, y' (or $\frac{dy}{dx}$), of each function. Make only obvious simplifications.

(a) $y = x^4 e^{4-x^2}$

$$y' = x^4 e^{4-x^2}(-2x) + 4x^3 e^{4-x^2}$$
$$= 2x^3 e^{4-x^2}(-x^2 + 2)$$
$$= 2(2-x^2)x^3 e^{4-x^2}$$

(b) $y = x^2 \sin\left(\frac{1}{x^2}\right)$

$$y' = x^2 \cos\frac{1}{x^2}\left(-\frac{2}{x^3}\right) + 2x \sin\left(\frac{1}{x^2}\right)$$
$$= -\frac{2}{x}\cos\frac{1}{x^2} + 2x \sin\left(\frac{1}{x^2}\right)$$

(c) $y = \ln(1 - \tan^2(3x))$

$$y' = \frac{-2\tan 3x \sec^2 3x \cdot 3}{1 - \tan^2 3x} = \frac{-6\tan 3x \sec^2 3x}{1 - \tan^2 3x}$$

4 2. Using logarithmic differentiation, find $f'(x)$ if $f(x) = (\cos x)^{2x}$

$$\ln f(x) = 2x \ln \cos x$$

$$\frac{f'(x)}{f(x)} = 2x\left(-\tan x\right) + 2\ln\cos x$$

$$= -2x\tan x + 2\ln\cos x$$

$$f'(x) = (\cos x)^{2x}\left(2\ln\cos x - 2x\tan x\right)$$

6 3. Find the equation of the **tangent** line to the curve $2x^3 = 9xy - 2y^3$ at the point $(2, 1)$.

$$6x^2 = 9xy' + 9y - 6y^2y'$$

$$24 = 18y' + 9 - 6y'$$

$$15 = 12y'$$

$$y' = \frac{15}{12} = \frac{5}{4}$$

$$y - 1 = \frac{5}{4}(x - 2) \qquad y = \frac{5}{4}x - \frac{3}{2}.$$

7 4. A pebble dropped into a pond causes a circular ripple. Find the rate of change of the radius of the ripple at the instant in time when the radius is 1 metre and the area enclosed by the ripple is increasing at the rate of 4 square metres per second.

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

when $r=1$,

$4 = 2\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{2}{\pi}$

When $r=1$, the radius is increasing at $\frac{2}{\pi}$ m/sec.

5 5. Find the derivative and the critical numbers only for the function $y = \frac{x^2}{x-k}$. The number k is a constant.

$y' = \frac{(x-k)2x - x^2}{(x-k)^2} = \frac{x^2 - 2kx}{(x-k)^2} = \frac{x(x-2k)}{(x-k)^2}$

$x=0$, $x=k$, $x=2k$ are the crit. #'s.

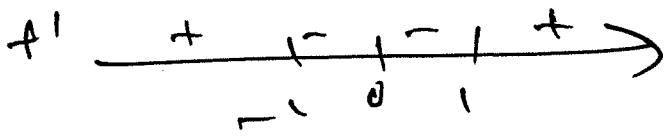
5 6. The function $y = xe^{bx}$ has one critical number where b is a nonzero constant. Find the critical number in terms of b.

$y' = x e^{bx} \cdot b + e^{bx} = e^{bx}(bx+1)$

$-\frac{1}{b}$ is the crit. #.

8 7. Find the intervals of monotonicity and local extrema of the function $f(x) = 3x^5 - 5x^3$. Show all work. Do **not** sketch the graph.

$f' = 15x^4 - 15x^2 = 15x^2(x^2-1)$



Increasing: $(-\infty,-1)$, $(1,\infty)$

Decreasing: $(-1,1)$ or $(-1,0)$, $(0,1)$

Local Max: $x=-1$

Local Min $x=1$.