SOLUTIONS

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH 1000 **QUIZ #1** – *Oct. 19, 2009* **NAME:**

Marks

15 1. Evaluate each limit using the limit rules. **No graphs or tables.**

(a) $\lim_{x\to 2} \frac{7x-6-2x^2}{3x^2-12} = \lim_{x\to 2} \frac{(2x-3)(2-x)}{3(x-2)(x+2)}$

$= -\lim_{x\to 2} \frac{2x-3}{3(x+2)}$

$= -\frac{1}{12}$

(b) $\lim_{x\to 1} \frac{x^3-1}{\sqrt{x}-1} = \lim_{x\to 1} \frac{(x-1)(x^2+x+1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)}$

$= \lim_{x\to 1} \frac{(x-1)(x^2+x+1)(\sqrt{x}+1)}{x-1}$

$= \lim_{x\to 1} (x^2+x+1)(\sqrt{x}+1)$

$= 6$

(c) $\lim_{x\to 0} f(x)$ where $f(x) = \begin{cases} \dfrac{x^2+1}{x+4} & \text{if } x<0 \\ 1 & \text{if } x=0 \\ \dfrac{\sin(x^2)}{4x^2} & \text{if } x>0. \end{cases}$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{1}{4}\cdot\frac{\sin x^2}{x^2} = \frac{1}{4}\cdot 1 = \frac{1}{4}$$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x^2+1}{x+4} = \frac{1}{4}$$

$$\boxed{\lim_{x\to 0} f(x) = \frac{1}{4}}$$

7 2. Find any vertical and horizontal asymptotes of the following function. Classify any vertical asymptotes. Justify with the appropriate infinite limits and limits at infinity.

$$f(x) = \frac{x^2+2}{4x^2-1}$$

$$f(x) = \frac{x^2+2}{(2x-1)(2x+1)}$$

Vertical Asymptotes

$x = \frac{1}{2}$ $x = -\frac{1}{2}$

f: $+$ | $-\frac{1}{2}$ | $-$ | $\frac{1}{2}$ | $+$

Classification

$$\lim_{x\to -\frac{1}{2}^-} f(x) = \infty \qquad \lim_{x\to -\frac{1}{2}^+} f(x) = -\infty$$

$x = -\frac{1}{2}$ odd

$$\lim_{x\to \frac{1}{2}^+} f(x) = \infty \qquad \lim_{x\to \frac{1}{2}^-} f(x) = -\infty$$

$x = \frac{1}{2}$ odd

$$\lim_{x\to \infty} \frac{x^2+2}{4x^2-1} = \frac{1}{4}$$

$y = \frac{1}{4}$ is a Horizontal Asymptote

8 3. Find and classify any discontinuities of $f(x)$ if $f(x) = \begin{cases} \dfrac{x^2+3}{x+1} & \text{if } x \le 0 \\ 2\cos x & \text{if } x > 0. \end{cases}$

Infinite discontinuity at $x = -1$. $\left(\lim_{x \to -1^-} f(x) = -\infty \right.$ and $\left. \lim_{x \to -1^+} f(x) = +\infty \right)$

$$\left.\begin{aligned} \lim_{x \to 0^-} f(x) &= \lim_{x \to 0^-} \frac{x^2+3}{x+1} = 3 \\ \lim_{x \to 0^+} f(x) &= \lim_{x \to 0^+} 2\cos x = 2 \end{aligned}\right\}$$ Jump discontinuity at $x = 0$.

8 4. (a) Give the precise mathematical definition of the derivative, $f'(x)$, of the function $f(x)$.

$f'(x) = \lim_{w \to x} \dfrac{f(w) - f(x)}{w - x}$ if the limit exists

(b) Using the **definition** of the derivative, find $f'(x)$ if $f(x) = 2x^2 - 3x$.

$$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$$

$$= \lim_{w \to x} \frac{(2w^2 - 3w) - (2x^2 - 3x)}{w - x}$$

$$= \lim_{w \to x} \frac{2(w-x)(w+x) - 3(w-x)}{w - x}$$

$$= \lim_{w \to x} \left(2(w+x) - 3 \right)$$

$$= 4x - 3.$$

8 5. Using the **rules** for derivatives, find the derivative, y' or $\frac{dy}{dx}$, of each function.

(a) $y = \frac{3x^2 - 2x + 1}{x + 1}$.

$$y' = \frac{(x+1)(6x-2) - (3x^2-2x+1)}{(x+1)^2}$$

$$= \frac{6x^2+4x-2-3x^2+2x-1}{(x+1)^2}$$

$$= \frac{3x^2+6x-3}{(x+1)^2} = \frac{3(x^2+2x-1)}{(x+1)^2}$$

(b) $y = \sin^3(\pi x)$

$$y' = 3\sin^2(\pi x) \cdot \cos(\pi x) \cdot \pi$$

$$= 3\pi \cos \pi x \sin^2 \pi x$$

4 6. Evaluate the derivative of the differentiable function $y = x^2 f(x^2)$ at $x = 2$ given that $f(4) = 3$ and $f'(4) = -1$.

$$y' = x^2 f'(x^2) \cdot 2x + 2x f(x^2)$$

At $x=2$,

$$y'(2) = 16(-1) + 4 \cdot 3$$

$$= \boxed{-4}$$