PRACTICE FINAL — W07 EXAM

1a $$\lim_{x\to 2} \frac{3-\sqrt{x+7}}{x-2} = \lim_{x\to 2} \frac{(3-\sqrt{x+7})(3+\sqrt{x+7})}{(x-2)(3+\sqrt{x+7})}$$

$$= \lim_{x\to 2} \frac{9-(x+7)}{(x-2)(3+\sqrt{x+7})} = \lim_{x\to 2} \frac{2-x}{(x-2)(3+\sqrt{x+7})}$$

$$= -\lim_{x\to 2} \frac{1}{3+\sqrt{x+7}} = -\frac{1}{6}.$$

b $$\lim_{x\to 0} \frac{\sin 6x}{x\cos^2 x} = 6 \lim_{x\to 0} \frac{\sin 6x}{6x} \cdot \lim_{x\to 0} \frac{1}{\cos^2 x}$$

$$= 6 \cdot 1 \cdot 1 = 6$$

c $$\lim_{x\to -\infty} \frac{3x+1}{\sqrt{4x^2-5}} = \lim_{x\to -\infty} \frac{x(3+\frac{1}{x})}{\sqrt{x^2(4-\frac{5}{x^2})}}$$

$$= \lim_{x\to -\infty} \frac{\cancel{x}(3+\frac{1}{x})}{-\cancel{x}\sqrt{4-5/x^2}} = -\lim_{x\to -\infty} \frac{3+\frac{1}{x}}{\sqrt{4-5/x^2}} = -\frac{3}{2}.$$

d $$\lim_{x\to -1^-} \frac{2x^2+x-1}{x^2+2x+1} = \lim_{x\to -1^-} \frac{(2x-1)(x+1)}{(x+1)^2}$$

$$= \lim_{x\to -1^-} \frac{2x-1}{x+1} = \infty.$$

x	y
-1.1	$\frac{-}{-} = +$

e $$\lim_{x\to 2^-} \frac{|x^2-4|}{x-2} = \lim_{x\to 2^-} \frac{4-x^2}{x-2}$$

$$= \lim_{x\to 2^-} \frac{(2-x)(2+x)}{x-2} = -\lim_{x\to 2^-} 2+x = -4.$$

2. $f'(x) = \lim_{w \to x} \dfrac{\sqrt{w+2} - \sqrt{x+2}}{w - x}$ if the limit exists

$= \lim_{w \to x} \dfrac{(w+2) - (x+2)}{(w-x)(\sqrt{w+2} + \sqrt{x+2})} = \lim_{w \to x} \dfrac{1}{\sqrt{w+2} + \sqrt{x+2}}$

$= \dfrac{1}{2\sqrt{x+2}}$.

3. $f(1) = 2$.

$$\left.\begin{aligned} \lim_{x \to 1^-} f(x) &= \lim_{x \to 1^-} x^2 + 1 = 2 \\ \lim_{x \to 1^+} f(x) &= \lim_{x \to 1^+} 3x^2 - 1 = 3 - 1 = 2 \end{aligned}\right\} L = 2.$$

$f(1) = L = 2$ Continuous at $x = 1$.

4a $y' = \dfrac{(1+\cos x)\cos x - (1+\sin x)(-\sin x)}{(1+\cos x)^2} = \dfrac{\cos x + \cos^2 x + \sin x + \sin^2 x}{(1+\cos x)^2}$

$= \dfrac{\sin x + \cos x + 1}{(1+\cos x)^2}$.

b $f'(x) = 2\tan(3x^4 - 5)\sec^2(3x^4 - 5)(12x^3)$

c $y' = -\dfrac{\sin(\ln x)}{x} + \dfrac{x(-\sin x) + \cos x}{x\cos x}$

d $\ln y = \ln 5^x + \ln \sin^4 x - \ln(3x^5 - 7x)^{1/2}$

$\ln y = x\ln 5 + 4\ln\sin x - \frac{1}{2}\ln(3x^5 - 7x)$

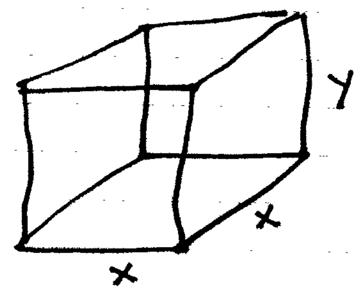
$\dfrac{y'}{y} = \ln 5 + 4\dfrac{\cos x}{\sin x} - \dfrac{1}{2}\dfrac{15x^4 - 7}{3x^5 - 7x}$

$y' = y(\qquad\qquad)$

5. $f'(x) = (x-1)e^x + e^x \qquad f'(1) = e$

Tangent: $y - 0 = e(x-1)$ or $y = ex - e$.

6. LET V REPRESENT THE VOLUME TO BE MAXIMIZED.



$V = x^2 y$

CONSTRAINT $x^2 + 4xy = 108$.

$y = \dfrac{108 - x^2}{4x} \qquad x > 0$.

$0 < x \le \sqrt{108}$.

$$V(x) = x^2\left(\frac{108 - x^2}{4x}\right)$$

$$= \frac{1}{4}x(108 - x^2)$$

$$= \frac{1}{4}(108x - x^3)$$

$$V'(x) = \frac{1}{4}(108 - 3x^2) = \frac{3}{4}(36 - x^2)$$

V': $+$ before 6, $-$ after, on the x-axis.

MAX VOLUME WHEN $x = 6$

DIMENSIONS: 6 BY 6 BY 3 (ALL METRES).

7. USE IMPLICIT DIFFERENTIATION.

$xy' + y + 1 - 2y' = 0$ or $y'(x-2) = -(1+y)$

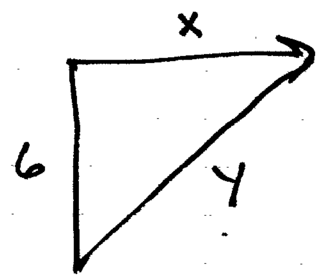
$$\therefore \quad y' = -\frac{y+1}{x-2}.$$

$$y'' = -\frac{(x-2)y' - (y+1)}{(x-2)^2} = -\frac{(x-2)\left(-\frac{y+1}{x-2}\right) - (y+1)}{(x-2)^2}$$

$$= \frac{y+1+y+1}{(x-2)^2} = \frac{2(y+1)}{(x-2)^2}.$$

8. $\dfrac{dy}{dt} = 400$

$\dfrac{dx}{dt} = ?$



$y^2 = x^2 + 6^2$

$2y\dfrac{dy}{dt} = 2x\dfrac{dx}{dt}$ or $\dfrac{dx}{dt} = \dfrac{y}{x}\dfrac{dy}{dt}$

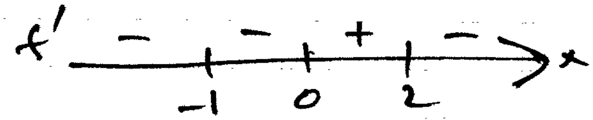
WHEN $y = 10$, $x = 8$

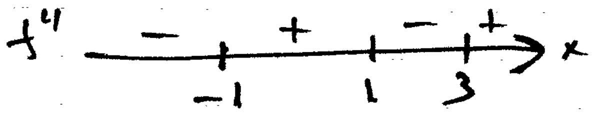
$\therefore \dfrac{dx}{dt} = \dfrac{10}{8}(400) = 500$

PLANE MOVES AT 500 km/hr.

9.

x	y
0	1
1	2
2	4
3	1
-2	0

f'


f''


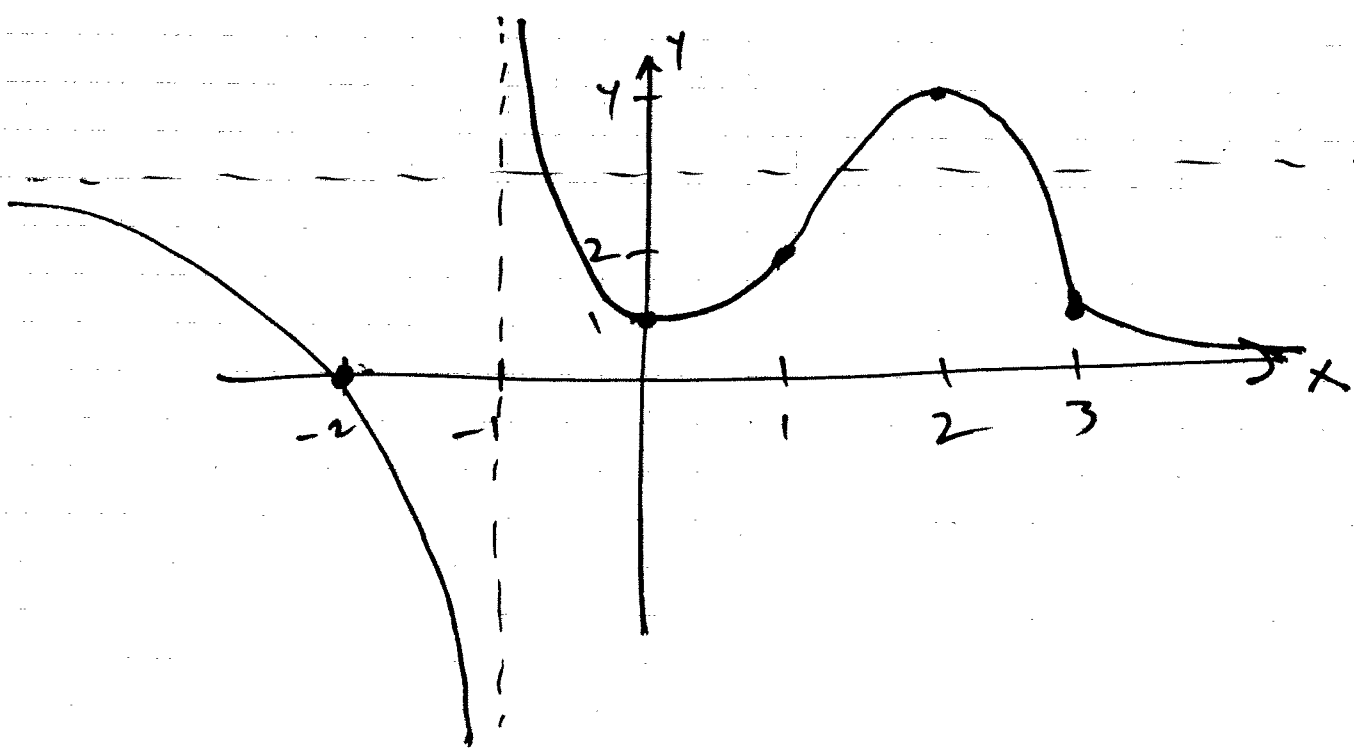
LOCAL MIN $x=0$

LOCAL MAX $x=2$

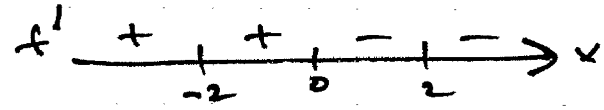
FLEX POINTS $x=1$, $x=3$.

$x = -1$, VERT. ASYMP.

$y = 3$, $y = 0$ HORIZ. ASYMP.



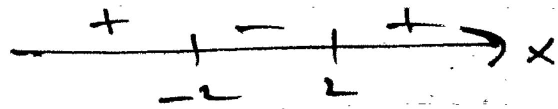
10 a)



LOCAL MAX AT $x=0$.

INCREASING: $(-\infty,-2)$ $(-2,0)$, DECREASING: $(0,2)$ $(2,\infty)$.

b)



CONCAVE UP: $(-\infty,-2)$, $(2,\infty)$ DOWN: $(-2,2)$.

NO FLEX POINTS.

11 a $\displaystyle\int \left(\sin(3x-1)+\frac{1}{x}\right)dx = -\frac{\cos(3x-1)}{3}+\ln|x|+C$

b $\displaystyle\int_{-1}^{1} 3x^2+1\,dx = (x^3+x)\Big|_{-1}^{1} = (1+1)-(-1-1) = 4.$

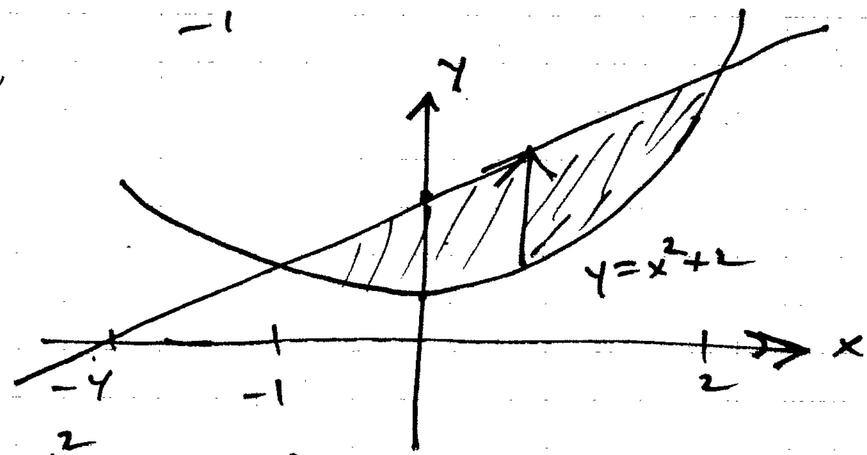
12. POINTS OF INTERSECTION

$x^2+2 = x+4$

$x^2-x-2=0$

$(x+1)(x-2)=0$

$x=-1,\ x=2.$



$$\int_{-1}^{2} (x+4)-(x^2+2)\,dx = \int_{-1}^{2} x+2-x^2\,dx$$

$$= \left(\tfrac{1}{2}x^2+2x-\tfrac{1}{3}x^3\right)\Big|_{-1}^{2} = \left(2+4-\tfrac{8}{3}\right)-\left(\tfrac{1}{2}-2+\tfrac{1}{3}\right)=\tfrac{9}{2}.$$

AREA IS $9/2$ SQ. UNITS.

13 a $\displaystyle f'(x)=\lim_{w\to x}\frac{f(w)-f(x)}{w-x} = \lim_{w\to x}\frac{\frac{1}{g(w)}-\frac{1}{g(x)}}{w-x}$

$$=\lim_{w\to x}\frac{g(x)-g(w)}{(w-x)(g(w)g(x))} = -\lim_{w\to x}\frac{g(w)-g(x)}{w-x}\cdot\lim_{w\to x}\frac{1}{g(w)g(x)}$$

$$= -g'(x)\,\frac{1}{(g(x))^2}.$$

b $f'(x) = 3x^2+4x+C$. $f'(-1)=2$ gives $C=3$.

$f(x) = x^3+2x^2+3x+D$. $f(-1)=1$ gives $D=3$.