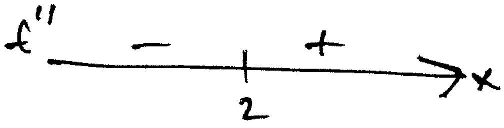
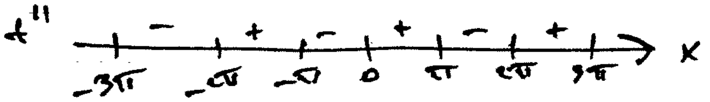
Mivao Solutions to Worksheet # 9 F09.

1a $f'(x) = 3x^2 - 12x + 9$
$f''(x) = 6x - 12 = 6(x-2)$



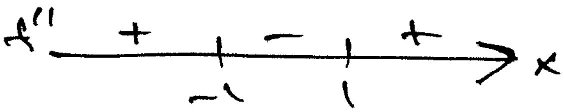
Concave Down on $(-\infty, 2)$. Concave up on $(2, \infty)$. $f(x)$ is continuous at $x=2$ so $(2, f(2)) = (2,1)$ is a point of inflection.

1b $f'(x) = 1 - 2\cos x$ $f''(x) = 2\sin x$.
$f''(x) = 0$ at $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi$.



Concave up: $(-2\pi, -\pi)$, $(0, \pi)$ and $(2\pi, 3\pi)$.
Concave Down: $(-3\pi, -2\pi)$, $(-\pi, 0)$, $(\pi, 2\pi)$.
Points of Inflection at $x = 0, \pm\pi, \pm 2\pi$.

1c $f'(x) = 4x^3 - 12x$
$f''(x) = 12x^2 - 12 = 12(x^2-1)$

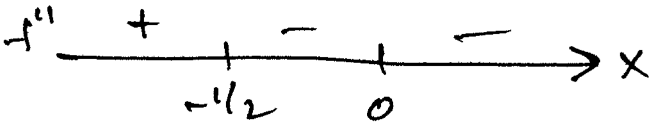


Concave Up: $(-\infty, -1)$, $(1, \infty)$ Concave Down $(-1, 1)$.
Points of Inflection at $x = \pm 1$.

1d $f'(x) = x^{2/3}(-2) + \frac{2}{3}x^{-1/3}(5-2x) = \frac{1}{3}x^{-1/3}(-6x + 2(5-2x))$
$= \frac{2}{3}\frac{(-5x+5)}{x^{1/3}} = \frac{-10}{3}\frac{(x-1)}{x^{1/3}}$.

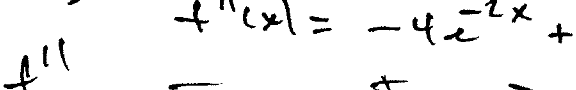
$$f''(x) = -\frac{10}{3}\frac{x^{1/3} - (x-1)\frac{1}{3}x^{-2/3}}{x^{2/3}} = -\frac{10}{9}\frac{x^{-2/3}(3x - (x-1))}{x^{2/3}}$$

$$= -\frac{10}{9}\frac{2x+1}{x^{4/3}}.$$



Concave Up: $(-\infty, -\frac{1}{2})$. Concave Down: $(-\frac{1}{2}, 0)$, $(0, \infty)$.
Point of Inflection at $x = -1/2$.

1e $f'(x) = -4xe^{-2x} + 2e^{-2x} = -2e^{-2x}(2x-1)$
$f''(x) = -4e^{-2x} + 4e^{-2x}(2x-1) = -4e^{-2x}(1-(2x-1)) = -4e^{-2x}(2-2x)$

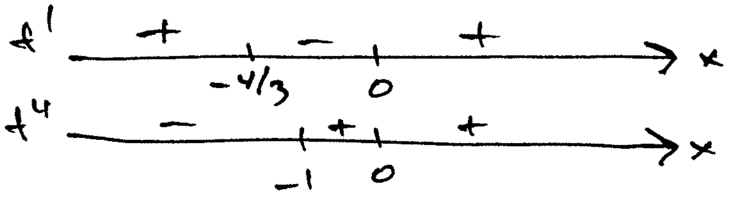


Concave Down: $(-\infty, 1)$. Concave Up $(1, \infty)$

1e (Cont'd). Point of Inflection at $x=1$.

2a $f'(x) = 15x^4 + 20x^3 = 5x^3(3x+4)$

$f''(x) = 60x^3 + 60x^2 = 60x^2(x+1)$



Increasing: $(-\infty, -4/3)$, $(0, \infty)$

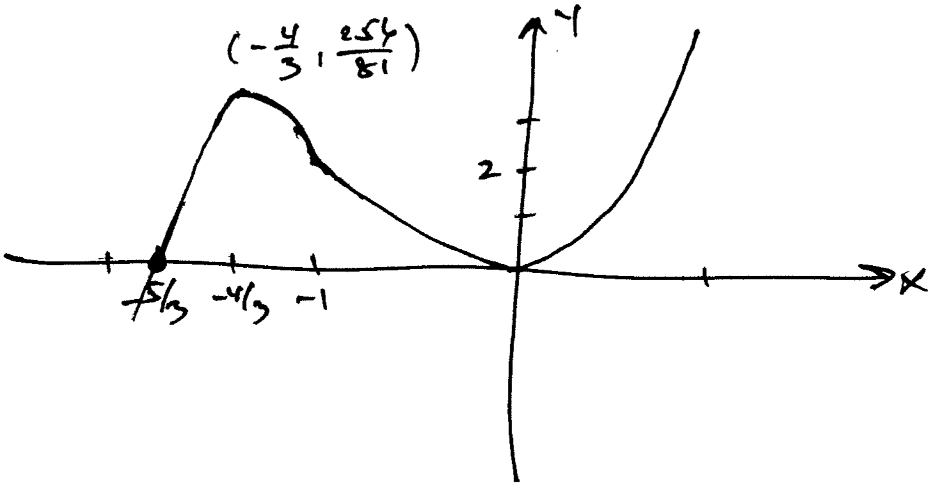
Decreasing: $(-4/3, 0)$

$f(x) = x^4(3x+5)$

Concave Down: $(-\infty, -1)$,

Concave Up: $(-1, 0)$, $(0, \infty)$.

x	y
$-5/3$	0
$-\frac{4}{3}$	$\frac{256}{81}$
-1	2
0	0

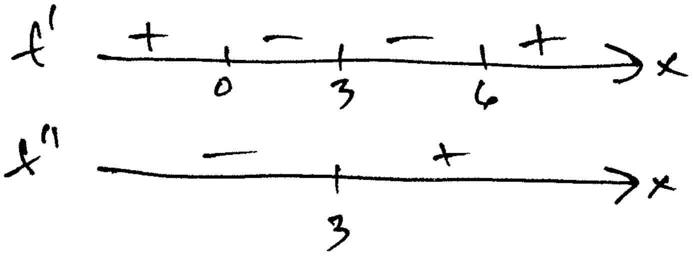


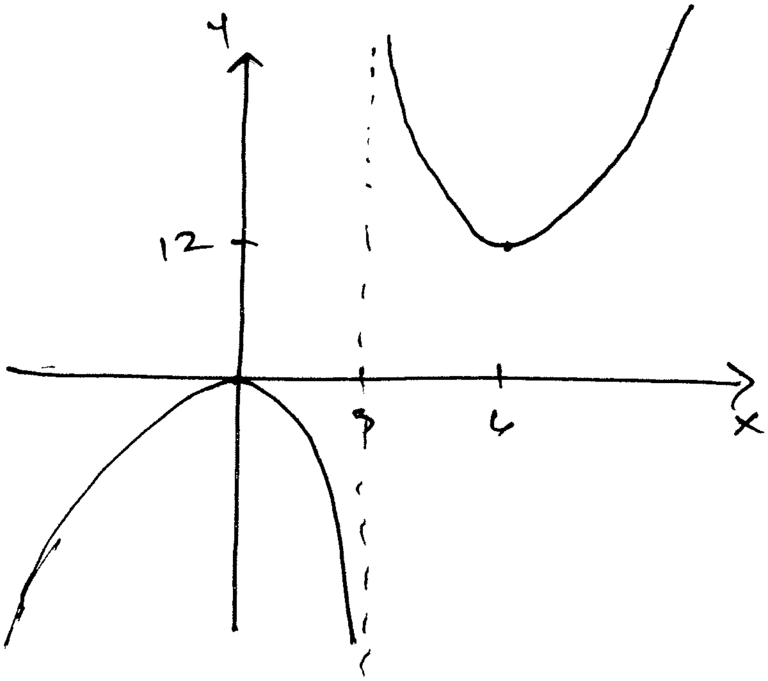
2b $f'(x) = \frac{(x-3)2x - x^2}{(x-3)^2} = \frac{x^2-6x}{(x-3)^2} = \frac{x(x-6)}{(x-3)^2}$

$$f''(x) = \frac{(x-3)^2(2x-6) - x(x-6)2(x-3)}{(x-3)^4} = \frac{2(x-3)((x-3)^2 - x^2 + 6x)}{(x-3)^4}$$

$$= \frac{2(x^2-6x+9-x^2+6x)}{(x-3)^3}$$

$$= \frac{18}{(x-3)^3}.$$

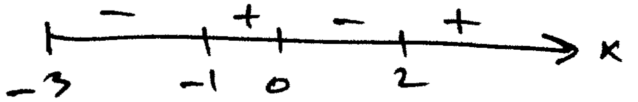


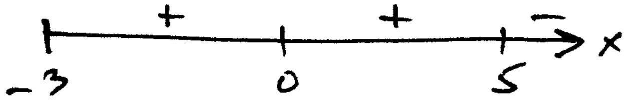


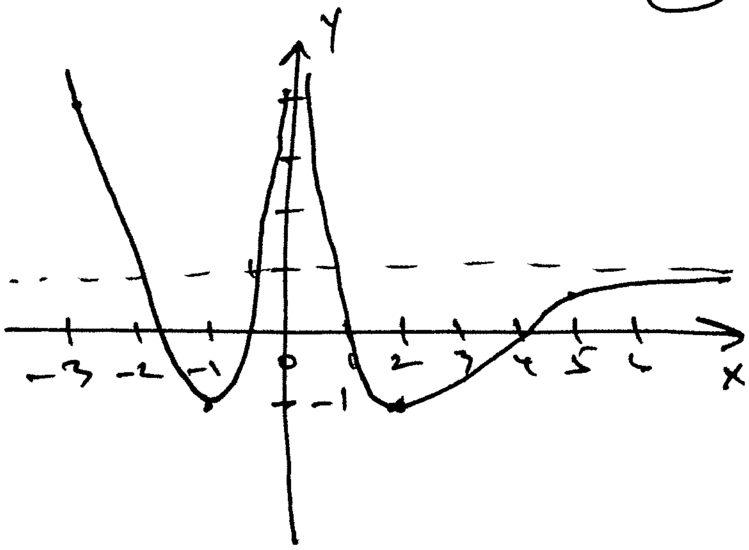
$x=3$ is a Vertical Asymptote

No Horizontal Asymptote

3





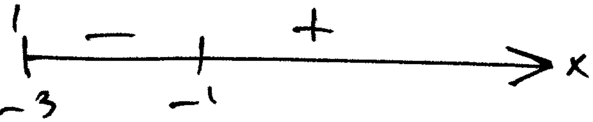


y = 1 Horizontal Asymptote
x = 0 Vertical Asymptote

x	−3	−1	0	2	5
y	4	−1	−1	−1	?.

(Intercepts are not given. f(5) is not given either (Put it ~ 0.5)
x = 5 is a point of inflection.

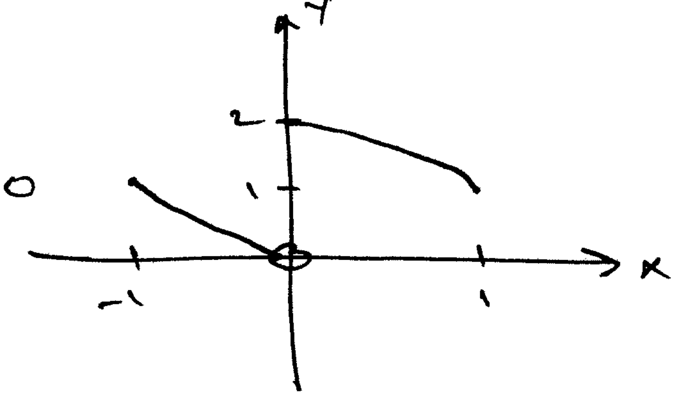
4a $f'(x) = xe^x + e^x = e^x(x+1)$



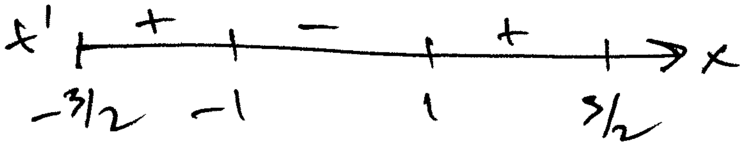
$\lim_{x\to\infty} xe^x = \infty$. No Absolute Maximum. The Absolute Minimum occurs at $x=-1$, Min is $-\frac{1}{e}$.

4b $f(x) = \begin{cases} x^2 & \text{on } [-1,0) \\ 2-x^2 & \text{on } [0,1]. \end{cases}$

Absolute Max is 2. Occurs at x = 0
No Absolute Minimum.



4c $f'(x) = 10x^4 - 10 = 10(x^4-1) = 10(x^2-1)(x^2+1)$

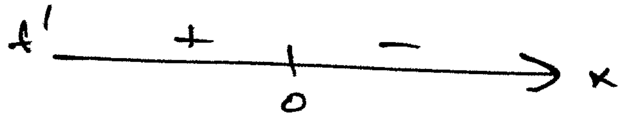


x	−3/2	−1	1	3/2
y	$\frac{77}{16}$	13	−3	$\frac{83}{16}$

Absolute Max: 13.
Absolute Min: −3.

$D_f = [-3/2, 3/2]$.

4d $f'(x) = \frac{-2x}{(1+x^2)^2}$

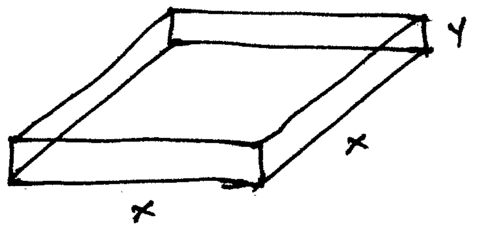


Absolute Maximum: 1, (At $x=0$)

No Absolute Min. $\lim_{x\to\infty} \frac{1}{1+x^2} = 0 = \lim_{x\to-\infty} \frac{1}{1+x^2}$.

But 0 is not a y-value for the function. ∴ no smallest y-coordinate.

5 Let V represent the volume to be maximized.



$V = x \cdot x \cdot y = x^2 y$

$V = x^2\left(\frac{108-x^2}{4x}\right)$

$= \frac{1}{4}x(108-x^2)$

$V' = \frac{1}{4}x(-2x) + \frac{1}{4}(108-x^2)$

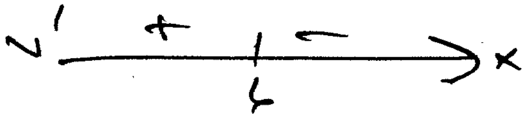
$= \frac{1}{4}(-2x^2+108-x^2) = \frac{1}{4}(108-3x^2)$

$= \frac{3}{4}(36-x^2)$

Auxiliary Equation

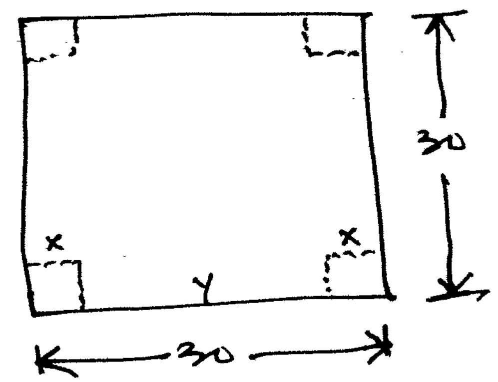
$4xy + x^2 = 108.$

$y = \frac{108-x^2}{4x}.$



Max. Volume Occurs when dimensions are 6×6×3 cm.

6 Let V represent the volume to be maximized.



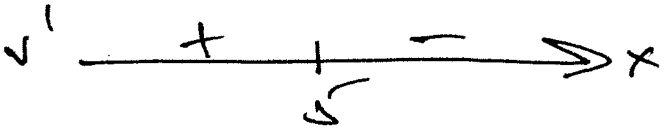
$V = (30-2x)^2 \cdot x$

$= 4x(15-x)^2 \qquad 0<x<15.$

$V' = -8x(15-x) + 4(15-x)^2$

$= -4(15-x)(2x-(15-x))$

$= -4(15-x)(3x-15)$



Auxiliary Equation

$2x+y=30$

$y = 30-2x$

Dimensions of the box of largest volume are: 20×20×5 cm.