M1000 Solutions to Worksheet #8 F09.

1. Use $s = -\frac{g}{2}t^2 + v_0t + s_0$ with $g = 32$, $v_0 = 132$, $s_0 = 6$.

NOTE: 1 MILE = 5280 FEET. $\boxed{90 \text{ mi/hr} = 132 \text{ ft/sec.}}$

$$s = -16t^2 + 132t + 6.$$

Baseball reaches its highest point when $v = 0$.

$v = -32t + 132 = 0$ if $t = \frac{132}{32} = \frac{33}{8}$ sec.

$s(\frac{33}{8}) = 278.25$ feet.

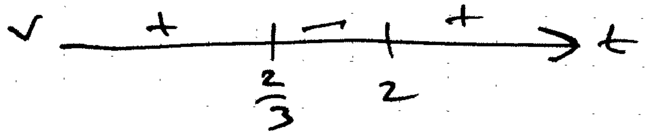
Baseball will <u>not</u> hit the roof. It will come within 31.75 feet.

2. <u>a</u> $v = 3t^2 - 8t + 4 = (3t-2)(t-2)$

$a = 6t - 8$

<u>b</u> object is at rest after $\frac{2}{3}$ sec and after 2 sec.

<u>c</u>



Moving right for $0 \le t < 2/3$ and $t > 2$

Moving left for $\frac{2}{3} < t < 2$.

<u>d</u> $a(\frac{2}{3}) = 6(\frac{2}{3}) - 8 = -4$ cm/sec²

$a(2) = 12 - 8 = 4$ cm/sec².

<u>e</u> Total distance $= |s(\frac{2}{3}) - s(0)| + |s(2) - s(\frac{2}{3})| + |s(4) - s(2)|$

$= |\frac{32}{27} - 0| + |0 - \frac{32}{27}| + |16 - 0| = \frac{64}{27} + 16$

$= \frac{496}{27} \approx 18.4$ cm.

3. $v = s' = 4k\cos(kt+c) + 3k\sin(kt+d)$

$a = v' = s'' = -4k^2\sin(kt+c) + 3k^2\cos(kt+d)$

$= -k^2(4\sin(kt+c) - 3\cos(kt+d))$

$\therefore\ a = -k^2 s$. (This denotes s.h.m.)

4a $f'(x) = 3x^2 - 12x + 9 = 3(x^2-4x+3) = 3(x-3)(x-1)$.

f' + | − | + → x

1 3

Monotone increasing $(-\infty,1), (3,\infty)$

decreasing on $(1,3)$.

$x=1$ gives a local maximum. $x=3$ a local minimum

4b $f'(x) = \dfrac{(3x^2-1)2x - (x^2-3)6x}{(3x^2-1)^2} = \dfrac{2x(3x^2-1-3(x^2-3))}{(3x^2-1)^2}$

$= \dfrac{2x(3x^2-1-3x^2+9)}{(3x^2-1)^2} = \dfrac{16x}{(3x^2-1)^2}$.

f' − | − | + | + → x

$-\frac{1}{\sqrt{3}}$ 0 $\frac{1}{\sqrt{3}}$

Decreasing $(-\infty,-\frac{1}{\sqrt{3}}), (-\frac{1}{\sqrt{3}},0)$

Increasing $(0,\frac{1}{\sqrt{3}}), (\frac{1}{\sqrt{3}},\infty)$.

Local Minimum at $x=0$. (Not continuous at $\pm\frac{1}{\sqrt{3}}$)

4c $y' = -\sin x - \cos x$. $y'=0$ if $\frac{\sin x}{\cos x} = -1$.

Critical Numbers in $[0,2\pi]$ are $x=\frac{3\pi}{4}$, $x=\frac{7\pi}{4}$.

y' − | + | − → x

$\frac{3\pi}{4}$ $\frac{7\pi}{4}$

Decreasing $[0,\frac{3\pi}{4}), (\frac{7\pi}{4},2\pi]$

Increasing $(\frac{3\pi}{4},\frac{7\pi}{4})$.

Local Minimum at $x=\frac{3\pi}{4}$. Local Maximum $x=\frac{7\pi}{4}$.

4d $f'(x) = x^3\,2(x+2) + 3x^2(x+2)^2 = x^2(x+2)(2x+3(x+2))$

$= x^2(x+2)(5x+6)$.

f' + | − | + | + → x

-2 $-4/5$ 0

Increasing: $(-\infty,-2), (-\frac{6}{5},\infty)$

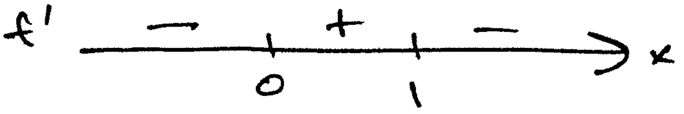
Decreasing: $(-2,-6/5)$.

Local Maximum at $x=-2$. Local minimum at $x=-6/5$.

4e $f'(x) = x^{2/3}(-2) + \frac{2}{3}x^{-1/3}(5-2x)$

$= -\frac{2}{3}x^{-1/3}\left(3x - (5-2x)\right) = -\frac{2}{3}\frac{5x-5}{x^{1/3}}$

$= -\frac{10}{3}\frac{(x-1)}{x^{1/3}} = \frac{10}{3}\frac{(1-x)}{x^{1/3}}$.

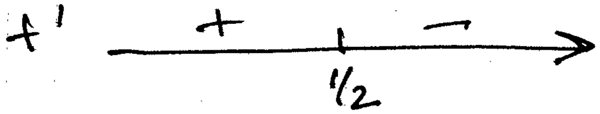


Decreasing: $(-\infty,0)$, $(1,\infty)$

Increasing: $(0,1)$.

Local Min at $x=0$. Local max at $x=1$.

4f $f'(x) = -4xe^{-2x} + 2e^{-2x} = 2e^{-2x}(-2x+1)$



Increasing: $(-\infty,\frac{1}{2})$

Decreasing: $(\frac{1}{2},\infty)$.

Local Max at $x = 1/2$.