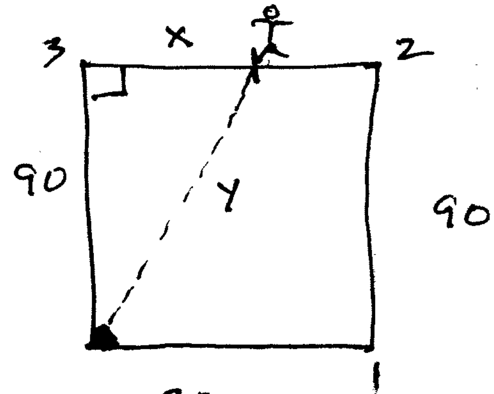
M1000 Solutions to Worksheet #7 F09

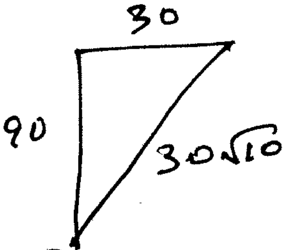
1. $\frac{dx}{dt} = -28$

$\frac{dy}{dt} = ?$ when $x = 30$.



$y^2 = x^2 + 90^2$

$\cancel{2}y \frac{dy}{dt} = \cancel{2}x \frac{dx}{dt}$

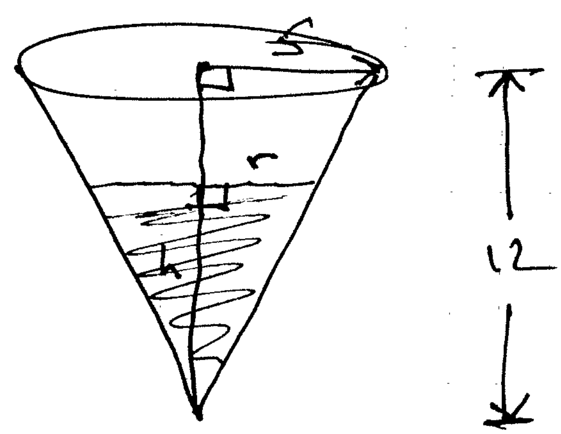


when $x = 30$, $y = 30\sqrt{10}$

$\therefore \frac{dy}{dt} = \frac{30}{30\sqrt{10}}(-28) = -\frac{28}{\sqrt{10}} = -\frac{28\sqrt{10}}{10} = -2.8\sqrt{10}$

when she is 30 feet from 3rd, y is decreasing at the rate of $28/\sqrt{10}$ ft/sec ($\simeq 8.9$ ft/sec).

2. $\frac{dV}{dt} = 10$ $\frac{dh}{dt} = ?$ when $h = 8$



Volume of water in tank is given by

$V = \frac{1}{3}\pi r^2 h$.

$V = \frac{1}{3}\pi\left(\frac{5h}{12}\right)^2 h = \frac{25\pi}{3 \cdot 12^2} h^3$

$\frac{5}{12} = \frac{r}{h}$

Similar triangles

$\boxed{r = \frac{5h}{12}}$

$\frac{dV}{dt} = \frac{25\pi}{12^2} h^2 \cdot \frac{dh}{dt}$

when $h = 8$,

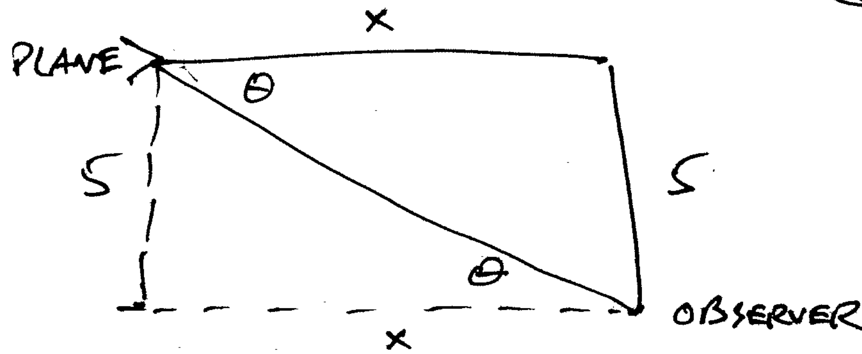
$\frac{dh}{dt} = \frac{1440}{25 \cdot 64\pi}$

Height is increasing at the approximate rate of 0.29 ft/min.

3.

$\frac{dx}{dt} = -600$

$\frac{d\theta}{dt}$ WHEN $\theta = \frac{\pi}{4}$?



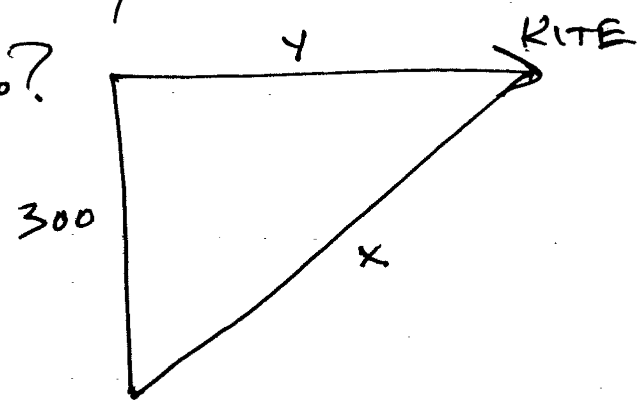
$\tan\theta = \frac{5}{x}$ so $\sec^2\theta \frac{d\theta}{dt} = -\frac{5}{x^2}\frac{dx}{dt}$

WHEN $\theta = \pi/4$, $\sec\theta = \frac{2}{\sqrt{2}} = \sqrt{2}$, $\tan\theta = 1$, $\underline{x = 5}$.

$\therefore \frac{d\theta}{dt} = \frac{1}{2}\left(\frac{-5}{5^2}\right)(-600) = \frac{600}{10} = 60$.

θ is increasing at 60 rad/hr.

4. $\frac{dx}{dt} = 20$ $\frac{dy}{dt} = ?$ $x = 500$?



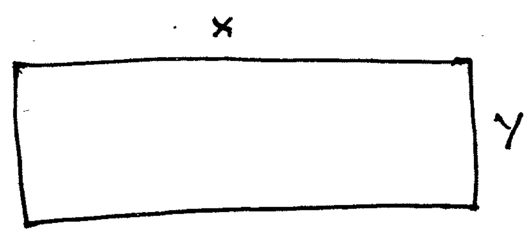
$x^2 = y^2 + 300^2$

$\cancel{2}x\frac{dx}{dt} = \cancel{2}y\frac{dy}{dt}$

WHEN $x = 500$, $y = 400$

$\frac{dy}{dt} = \frac{500}{400}(20) = 25$. Wind velocity is 25 ft/sec.

5. $\frac{dx}{dt} = 2$ $\frac{dy}{dt} = -1$.



when $x = 4$, $y = 1.5$

(a) $A = xy$. $\frac{dA}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt} = 4(-1) + (1.5)(2) = -1$.

Area is decreasing at 1 m²/sec.

(b) $P = 2x + 2y$ so, $\frac{dP}{dt} = 2\frac{dx}{dt} + 2\frac{dy}{dt} = 4 - 2 = 2$.

Perimeter increases at 2 m/sec.

(c) $d^2 = x^2 + y^2$ so, $\cancel{2}d\frac{dd}{dt} = \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}$.

when $x=4$, $y=1.5$, $d=\sqrt{73}/2$.

$$\therefore \frac{dd}{dt} = \frac{4(2)+1.5(-1)}{\sqrt{73}/2} = \frac{13/2}{\sqrt{73}/2} = \frac{13}{\sqrt{73}}$$

Diagonal is increasing at approx 1.5 m/sec.

6. $3\log_b x - \frac{1}{2}\log_b(1+x) + \log_b(1+x^2) = \log_b\left(\frac{x^3(1+x^2)}{\sqrt{1+x}}\right)$

7. $\log_b(2^x \tan x) = \log_b 2^x + \log_b \tan x = x\log_b 2 + \log_b \tan x$

8. a $\log_5(3x+1)(x+1) = 1$ so $(3x+1)(x+1) = 5$.

$\therefore 3x^2+4x-4=0$ or $(3x-2)(x+2)=0$

$x = 2/3$ is the only solution. Because $\log_5(x+1)$ is not defined when $x=-2$.

b $4^{x-2} = (2^x)^3$ means $(2^2)^{x-2} = 2^{3x}$

or $2^{2x-4} = 2^{3x}$ so $2x-4=3x$ or $x=-4$

9a $f(x) = 2\ln\sin x + \frac{3}{2}\ln(1+2x^3)$.

$f'(x) = 2\frac{\cos x}{\sin x} + \frac{3}{2}\frac{6x^2}{1+x^3} = 2\cot x + \frac{9x^2}{1+x^3}$.

b $f(x) = 3\ln(1-x) + \ln(1-3x) - 5\ln(x^2+2)$.

$f'(x) = \frac{-3}{1-x} + \frac{-3}{1-3x} - \frac{10x}{x^2+2}$

10a $\ln y = \ln x + \ln(x-1) + \ln(x-2) - 3\ln(x-3)$.

$y'/y = \frac{1}{x} + \frac{1}{x-1} + \frac{1}{x-2} - \frac{3}{x-3}$

$y' = y\left(\frac{1}{x} + \frac{1}{x-1} + \frac{1}{x-2} - \frac{3}{x-3}\right)$.

10b

$$\ln y = 2\ln\sin x + 4\ln\tan x - 2\ln(x^2+1)$$

$$y'/y = 2\cot x + 4\,\frac{\sec^2 x}{\tan x} - \frac{4x}{x^2+1}.$$

$$y' = y\left(2\cot x + \frac{4\sec^2 x}{\tan x} - \frac{4x}{x^2+1}\right).$$

10c

$$\ln y = \cos x \ln x$$

$$y'/y = \cos x \cdot \frac{1}{x} - \sin x \ln x$$

$$y' = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \ln x\right)$$

10d

$$\ln y = x^2 \ln(\ln x)$$

$$y'/y = x^2\,\frac{1/x}{\ln x} + 2x\ln(\ln x)$$

$$y' = (\ln x)^{x^2}\, x\left(\frac{1}{\ln x} + 2\ln(\ln x)\right).$$