M1000 Solutions to Worksheet #6 F09

1a $y = \ln(1-x^2)^4 = 4\ln(1-x^2)$. So, $y' = 4\,\frac{-2x}{1-x^2} = -\frac{8x}{1-x^2}$

b $y' = e^{\tan^2(\frac{1}{x})}\, 2\tan(\tfrac{1}{x})\sec^2(\tfrac{1}{x})\left(-\frac{1}{x^2}\right)$

c $y' = 4\sin^3\left(-\frac{3x}{4}\right)\cos\left(-\frac{3x}{4}\right)\left(-\frac{3}{4}\right) = -3\cos\left(-\frac{3x}{4}\right)\sin^3\left(-\frac{3x}{4}\right)$.

d $y' = 3\cot^2\left((9-x^2)^{1/2}\right)\cdot\left(-\csc^2(9-x^2)^{1/2}\right)\frac{1}{2}(9-x^2)^{-1/2}(-2x)$

$$= +\frac{3x\cot^2(9-x^2)^{1/2}\csc^2(9-x^2)^{1/2}}{(9-x^2)^{1/2}}$$

2. $y' = 2\sin(ax)\cos(ax)\cdot a - 2\cos(ax)(-\sin(ax))\cdot a$

$= 2a(\sin(ax)\cos(ax) + \sin(ax)\cos(ax))$

$= 2a(2\sin(ax)\cos(ax)) = 2a\sin(2ax)$

Double Angle formula

3. $y' = f(x)g'(x) + g(x)f'(x)$.

$y'' = f(x)g''(x) + g'(x)f'(x) + g'(x)f'(x) + g(x)f''(x)$

$y'' = f(x)g''(x) + 2f'(x)g'(x) + g(x)f''(x)$

$y''(2) = (-3)(2) + 2(-1)(-4) + (3)(2) = 8$.

4a $y' = 6x^2 - 2\sqrt{5}\,x + \sqrt{2}$

$y'' = 12x - 2\sqrt{5}$

b $y' = \sec^2(2x)(2) = 2\sec^2(2x)$

$y'' = 4\sec(2x)\cdot\sec 2x\tan 2x\cdot 2$

$= 8\tan 2x\sec^2 2x$.

c $y' = \frac{a}{1+ax}$ $\quad y'' = -\frac{a^2}{(1+ax)^2}$ $\quad y''' = \frac{2a^3}{(1+ax)^3}$.

4d $y' = x^2 e^{x^2} 2x + 2x e^{x^2} = 2x e^{x^2}(1+x^2)$

$$y'' = 2x e^{x^2}(2x) + (1+x^2)\left(2x e^{x^2} 2x + 2 e^{x^2}\right)$$
$$= e^{x^2}\left(4x^2 + (1+x^2)4x^2 + 2(1+x^2)\right)$$
$$= 2e^{x^2}(2x^2 + 2x^2 + 2x^4 + 1 + x^2)$$
$$= 2e^{x^2}(2x^4 + 5x^2 + 1)$$

5a $y' = 4(1-x^2)^3(-2x) = -8x(1-x^2)^3$

$$y'' = -8x\, 3(1-x^2)^2(-2x) - 8(1-x^2)^3$$
$$= -8(1-x^2)^2\left((1-x^2) - 6x^2\right)$$
$$= -8(1-7x^2)(1-x^2)^2 \quad \text{or} \quad 8(7x^2-1)(1-x^2)^2$$

5b $y' = 5\left(x+\frac{1}{x}\right)^4\left(1-\frac{1}{x^2}\right) = 5\left(1-\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)^4$

$$y'' = 5\left(1-\frac{1}{x^2}\right)4\left(x+\frac{1}{x}\right)^3\left(1-\frac{1}{x^2}\right) + \left(x+\frac{1}{x}\right)^4\left(\frac{10}{x^3}\right)$$
$$= \left(x+\frac{1}{x}\right)^3\left(20\left(1-\frac{1}{x^2}\right) + \left(x+\frac{1}{x}\right)\frac{10}{x^3}\right)$$

or $y'' = \left(x+\frac{1}{x}\right)^3\left(20 - \frac{10}{x^2} + \frac{10}{x^4}\right)$

5c $y = x(1-x)^{-2}$ $\quad y' = x(-2)(1-x)^{-3}(-1) + (1-x)^{-2}$

$$= (1-x)^{-3}\left(2x + (1-x)\right) = \frac{1+x}{(1-x)^3}.$$

$$y'' = \frac{(1-x)^3 - (1+x)3(1-x)^2(-1)}{(1-x)^6} = \frac{(1-x)^2\left((1-x)+3(1+x)\right)}{(1-x)^6}$$
$$= \frac{4+2x}{(1-x)^4} = \frac{2(2+x)}{(1-x)^4}.$$

5d $y' = \frac{1}{x} - \sin(1-x^3)(-3x^2)$

$y' = \frac{1}{x} + 3x^2 \sin(1-x^3)$

$y'' = -\frac{1}{x^2} + 3x^2 \cos(1-x^3)(-3x^2) + 6x \sin(1-x^3)$.

$= -1/x^2 - 9x^4 \cos(1-x^3) + 6x \sin(1-x^3)$.

6a $5y^4 y' + 3x^2 2yy' + 6xy^2 + 20x^3 = 0$

$y' = -\frac{6xy^2 + 20x^3}{5y^4 + 6x^2 y}$

6b $\cos(x-y)(1-y') = x\cos y \cdot y' + \sin y$

$\frac{\cos(x-y) - \sin y}{x\cos y + \cos(x-y)} = y'$.

7. $3(x+y)^2(1+y') = x^3 y' + 3x^2 y$.

At $(1,-2)$, $3(1)(1+y') = y' - 6$ or $2y' = -9$

or $y' = -9/2$.

Tangent: $y - (-2) = -\frac{9}{2}(x-1)$ or $y = -\frac{9}{2}x + \frac{5}{2}$

Normal: $y + 2 = \frac{2}{9}(x-1)$ or $y = \frac{2}{9}x - \frac{20}{9}$.

8 $xe^y y' + e^y = 2yy'$. At $(0,1)$

$e = 2y'$ or $y' = \frac{e}{2}$. Slope is $\frac{e}{2}$.