M1100 Solutions to Worksheet #5 F09.

1. $f'(x) = h'(g(x)) \cdot g'(x)$. So, $f'(2) = h'(g(2)) \cdot g'(2)$
$= h'(3) \cdot (-4) = 2(-4) = -8$.

2.a $f'(x) = 5(x^5 - 7x^3 + 3x - 6)^4 (5x^4 - 21x^2 + 3)$ GPR T2, R5

b $f'(x) = \dfrac{(b+x) - (a+x)}{(b+x)^2} = \dfrac{b-a}{(b+x)^2}$ Quotient

c $f'(x) = -\frac{1}{4}(x^4 + 4x)^{-5/4}(4x^3 + 4) = -\dfrac{x^3+1}{(x^4+4x)^{5/4}}$ GPR

d $f'(x) = \dfrac{(1+x^3)(-4x) - (1-2x^2)3x^2}{(1+x^3)^2} = \dfrac{-4x - 4x^4 - 3x^2 + 6x^4}{(1+x^3)^2}$
$= \dfrac{2x^4 - 3x^2 - 4x}{(1+x^3)^2} = \dfrac{x(2x^3 - 3x - 4)}{(1+x^3)^2}$ QUOTIENT

e $f'(x) = \sec^2(x^2) \cdot 2x + 2\tan x \sec^2 x$
$= 2(x\sec^2(x^2) + \tan x \sec^2 x)$. SUM

f $f'(x) = \sin(2x)(-\sin x) + \cos x \cos 2x \cdot 2$
$= 2\cos 2x \cdot \cos x - \sin 2x \sin x$. PRODUCT

g $y = e^{a^2+b^2x^3} \cdot (3b^2x^2)$ CHAIN

h $y = \dfrac{4x^3 - 3}{x^4 - 3x + 2}$ CHAIN

(i) $f(x) = (1+x)^{-5}$ $f'(x) = -5(1+x)^{-6} = -\dfrac{5}{(1+x)^6}$ GPR

j $f(x) = (1+x)^{-m}(2+x)^{-n}$. $f'(x) = (1+x)^{-m}(-n)(2+x)^{-n-1}$
$+ (2+x)^{-n}(-m)(1+x)^{-m-1} = (1+x)^{-m-1}(2+x)^{-n-1}(-n(1+x) - m(2+x))$
$= -\dfrac{n+2m + x(n+m)}{(1+x)^{m+1}(1+x)^{n+1}}$.

3. $y' = \frac{(2+\cos x)\cos x - \sin x(-\sin x)}{(2+\cos x)^2} = \frac{2\cos x + \cos^2 x + \sin^2 x}{(2+\cos x)^2}$

$= \frac{1+2\cos x}{(2+\cos x)^2}$. $y' = 0$ when $\cos x = -1/2$ or when $x = \frac{2\pi}{3}$ or $x = \frac{4\pi}{3}$.

4. $y' = -\frac{8}{x^3} - 1 = 0$ when $8 + x^3 = 0$ or $x = -2$.

a $x = -2$ b $y - 3 = -9(x-1)$ or $y = -9x + 12$.

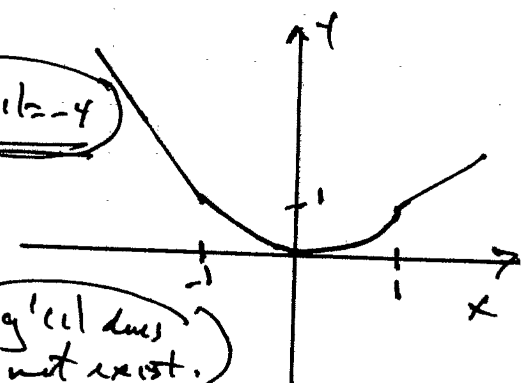
c $y - 3 = \frac{1}{9}(x-1)$ or $y = \frac{1}{9}x - \frac{1}{9} + 3 = \frac{x}{9} + \frac{26}{9}$.

5. $\lim_{w\to -1^-} \frac{g(w)-g(-1)}{w+1} = \lim_{w\to -1^-} \frac{(-3-4w)-1}{w+1} = -4$

$\lim_{w\to -1^+} \frac{g(w)-g(-1)}{w+1} = \lim_{w\to -1^+} \frac{w^4-1}{w+1} = -4$ $g'(-1) = -4$

$\lim_{w\to 1^+} \frac{g(w)-g(1)}{w-1} = \lim_{w\to 1^+} \frac{w-1}{w-1} = 1$

$\lim_{w\to 1^-} \frac{g(w)-g(1)}{w-1} = \lim_{w\to 1^-} \frac{w^4-1}{w-1} = 4$ $g'(1)$ does not exist.



g is differentiable at $x=-1$ but nondifferentiable at $x=1$.

6. $f'(x) = \frac{(x+1)2x - (x^2+3)}{(x+1)^2} = \frac{2x^2+2x-x^2-3}{(x+1)^2} = \frac{x^2+2x-3}{(x+1)^2}$

$= \frac{(x+3)(x-1)}{(x+1)^2}$. f': $+$ | $-$ | $-$ | $+$ at -3, -1, 1

$f'(x) > 0$ on $(-\infty, -3)$ and on $(1, \infty)$.

7. a $f'(x) = 2x + 4\left(x^3 + (x^4+x^2)^3\right)^3\left(3x^2 + 3(x^4+x^2)^2(4x^3+2x)\right)$

b $f'(1) = 218702$

c Coefficient of x^{34} is 5040