Math 1000 Solutions to worksheet #4 F09.

1 $f(x)$ is <u>continuous on $(-\infty,1)$</u> since $y=\sqrt{2-x}$ is continuous at each $c<1$.

On $(-\infty,2)$ $f(x)$ is continuous on $(-\infty,1)$ and on $(1,3)$. We need to check $c=1$, $f(1)=\sqrt{2-1}=1$. $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-}\sqrt{2-x}=1$ and $\lim\limits_{x\to 1^+} f(x)=\lim\limits_{x\to 1^+}\frac{x^3+1}{3-x}=\frac{2}{2}=1$. <u>Continuous on $(-\infty,2)$.</u>

<u>Continuous on $(-\infty,3)$</u>. Problem is at $x=3$ which is <u>not</u> in the interval $(-\infty,3)$.

$f(x)$ has a discontinuity at $x=3$ (infinite). Hence, $f(x)$ is <u>discontinuous on $(-\infty,4)$</u>.

2. Since $y=(x-1)^2$ and $y=1$ are continuous on $\mathbb{R}$ and $y=\ln x$ is continuous on $(0,\infty)$, the function $f(x)=(x-1)^2-\ln x-1$ is continuous on $(0,\infty)$. Hence, $f(x)$ is continuous on $[1,e]$. The Extreme Value Theorem applies on $[1,e]$. Since $f(1)=0-0-1=-1<0$ and $f(e)=(e-1)^2-2$ $=e^2-2e-1\approx 0.952>0$. Hence, the Location of Roots Theorem applies.

From the graph:

Abs Min $\approx$ [redacted] $f(1.364)\approx -1.17793$

Abs. Max $= f(e) = e^2-2e-1 \approx 0.952$

Zero ≈ 2.364 $(f(2.364)\approx 0)$.

Note: the absolute maximum of e^2-2e-1 is <u>exact</u>.

②

3a
$$\frac{f(w)-f(x)}{w-x} = \frac{(w^2-5w+1)-(x^2-5x+1)}{w-x}$$
$$= \frac{(w^2-x^2)-5(w-x)}{w-x}$$
$$= \frac{(w-x)(w+x)-5(w-x)}{w-x} = w+x-5 \quad (w\neq x$$
$$f'(x) = \lim_{w\to x}(w+x-5) = 2x-5.$$

3b
$$\frac{f(w)-f(x)}{w-x} = \frac{\frac{w-1}{1-2w}-\frac{x-1}{1-2x}}{w-x}$$
$$= \frac{\frac{(w-1)(1-2x)-(x-1)(1-2w)}{(1-2w)(1-2x)}}{w-x}$$
$$= \frac{w-\cancel{2wx}+2x\cancel{-1}-x+\cancel{2wx}-2w\cancel{+1}}{(1-2w)(1-2x)(w-x)}$$
$$= \frac{(w-x)-2(w-x)}{(1-2w)(1-2x)(w-x)} = \frac{-1}{(1-2w)(1-2x)} \qquad w\neq x.$$
$$f'(x) = \lim_{w\to x}\frac{-1}{(1-2w)(1-2x)} = -\frac{1}{(1-2x)^2}.$$

3c
$$\frac{f(w)-f(x)}{w-x} = \frac{\sqrt{3w-2}-\sqrt{3x-2}}{w-x}$$
$$= \frac{(\sqrt{3w-2}-\sqrt{3x-2})(\sqrt{3w-2}+\sqrt{3x-2})}{(w-x)(\sqrt{3w-2}+\sqrt{3x-2})}$$
$$= \frac{(3w-2)-(3x-2)}{(w-x)(\sqrt{3w-2}+\sqrt{3x-2})} = \frac{3(w-x)}{(w-x)(\sqrt{3w-2}+\sqrt{3x-2})}$$
$$= \frac{3}{\sqrt{3w-2}+\sqrt{3x-2}} \quad \text{if} \quad w\neq x.$$

3c (con't). $f'(x) = \lim_{w \to x} \dfrac{3}{\sqrt{3w-2} + \sqrt{3x-2}} = \dfrac{3}{2\sqrt{3x-2}}$.

4a $m_{tan} = f'(1) = 2 - 7 - 15 = -20$.

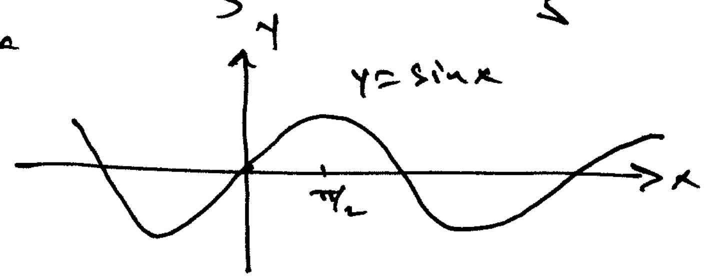
$m_{normal} = -\dfrac{1}{f'(1)} = \dfrac{1}{20}$.

4b Solve $f'(x) = 0$ or $2 - 7x - 15x^2 = 0$ for x.

$2 - 7x - 15x^2 = 0$ iff $(2+3x)(1-5x) = 0$

Horizontal Tangents at $x = -\frac{2}{3}$ and $x = \frac{1}{5}$.

5. At $x = \pi/2$, the tangent line should be horizontal. Tangent is $y = 1$.



6a T2, R2 $f'(x) = 9x^2 - 4x + 4$. $m_{tan} = f'(-1) = 17$.

6b T2, R2. $f'(x) = -2x^{-3} - e^x$. $m_{tan} = f'(-1) = 2 - \frac{1}{e}$.

6c T2, R1 $f'(x) = \frac{1}{5}(4 - 6x)$. $m_{tan} = f'(-1) = 2$.

7 T2, R2. $g'(x) = 6x^2 - 2x - 8 = 2(3x^2 - x - 4)$

$\therefore$ $g'(x) = 2(3x-4)(x+1)$. Horizontal tangents at each of $x = \frac{4}{3}$ and $x = -1$.

$g(\frac{4}{3}) = \frac{683}{27}$ so the horizontal tangent is $y = \frac{683}{27}$.

$g(-1) = 38$ so the horizontal tangent is $y = 38$.