Math 1000 Solutions to Worksheet #3 F09.

1a $f \quad \begin{array}{ccccc} + & | & - & | & + \\ & -2 & & 1 & \end{array} \rightarrow x$ $\qquad \lim\limits_{x\to -2^+} \dfrac{x^3-1}{x+2} = -\infty$. $x=-2$

1b $f \quad \begin{array}{ccc} - & | & + \\ & 3 & \end{array} \rightarrow x$ $\qquad \lim\limits_{x\to 3^-} \dfrac{e^x}{(x-3)^3} = -\infty$. $x=3$

2a $f(x) = \dfrac{6x^4+1}{(x^3-1)(3-2x)} = \dfrac{6x^4+1}{-2x^4+3x^3+2x-3}$ is a rational fn.

$\lim\limits_{x\to\infty} f(x) = \dfrac{6}{-2} = -3$. $y=-3$ is a horizontal asymptote.

2b $$\lim_{x\to\infty} \frac{\sqrt{4x^6-x}}{x^3+1} = \lim_{x\to\infty} \frac{\sqrt{x^6\left(4-\frac{1}{x^5}\right)}}{x^3(1+1/x^3)} = \lim_{x\to\infty} \frac{|x|^3\sqrt{4-1/x^5}}{x^3(1+1/x^3)}$$

$$= \lim_{x\to\infty} \frac{\sqrt{4-1/x^5}}{1+1/x^3} = 2.$$

$$\lim_{x\to-\infty} \frac{\sqrt{4x^6-x}}{x^3+1} = \lim_{x\to-\infty} \frac{|x|^3\sqrt{4-1/x^5}}{x^3(1+1/x^3)} = \lim_{x\to-\infty} \frac{-x^3\sqrt{4-1/x^5}}{x^3(1+1/x^3)}$$

$$= \lim_{x\to-\infty} -\frac{\sqrt{4-1/x^5}}{1+1/x^3} = -2. \qquad \boxed{y=2,\ y=-2}$$

3a $f \quad \begin{array}{ccccccc} + & | & - & | & + & | & - \\ & -2 & & 0 & & 2 & \end{array} \rightarrow x$ $\qquad f(x) = \dfrac{5x}{(2-x)(2+x)}$.

$\lim\limits_{x\to-2^-} f(x) = +\infty$, $\lim\limits_{x\to-2^+} f(x) = -\infty$ $\qquad x=-2$ odd

$\lim\limits_{x\to2^+} f(x) = -\infty$, $\lim\limits_{x\to2^-} f(x) = +\infty$ $\qquad x=2$ odd

$$\lim_{x\to\infty} \frac{5x}{4-x^2} = \lim_{x\to\infty} \frac{5x}{x^2\left(\frac{4}{x^2}-1\right)} = \lim_{x\to\infty} \frac{5}{x\left(\frac{4}{x^2}-1\right)} = 0.$$

$y=0$ Horizontal Asymptote.

3b $x^4+1 \geq 1$ for every x. Therefore there are <u>no</u> candidates for vertical asymptotes. But,

$$\lim_{x\to\infty} \frac{2x^4}{x^4+1} = 2$$ so $y=2$ is a horiz. asympt. (Rational functions have at most one asymptote).

3c $x^3+x^2-6x = x(x^2+x-6) = x(x+3)(x-2)$.

$$f \quad \begin{array}{ccccccc} - & | & + & | & - & | & + \\ & -3 & & 0 & & 2 & \end{array} \rightarrow x \qquad f(x) = \frac{1}{x(x+3)(x-2)}$$

$\lim_{x\to -3^-} f(x) = -\infty$, $\lim_{x\to -3^+} f(x) = +\infty$, $\lim_{x\to 0^-} f(x) = +\infty$,

$\lim_{x\to 0^+} f(x) = -\infty$, $\lim_{x\to 2^-} f(x) = -\infty$, $\lim_{x\to 2^+} f(x) = +\infty$.

$x=-3$, $x=0$, $x=2$ are <u>odd</u> vertical asymptotes.

$\lim_{x\to\infty} \frac{1}{x^3+x^2-6x} = 0$ $y=0$ is a horizontal asymptote.

3d $f(x) = \frac{x+4}{x^2-16} = \frac{x+4}{(x-4)(x+4)}$. Since $\lim_{x\to -4} \frac{x+4}{x^2-16}$

$= -\frac{1}{8}$, <u>the only infinite limit</u> is at $x=4$.

$$f \quad \begin{array}{ccccc} - & | & - & | & + \\ & -4 & & 4 & \end{array} \rightarrow x$$. $\lim_{x\to 4^-} f(x) = -\infty$, $\lim_{x\to 4^+} f(x) = +\infty$,

$x=4$ is an <u>odd</u> vertical asymptote.

$\lim_{x\to\infty} \frac{x+4}{x^2-16} = 0$. $y=0$ is a horizontal asymptote.

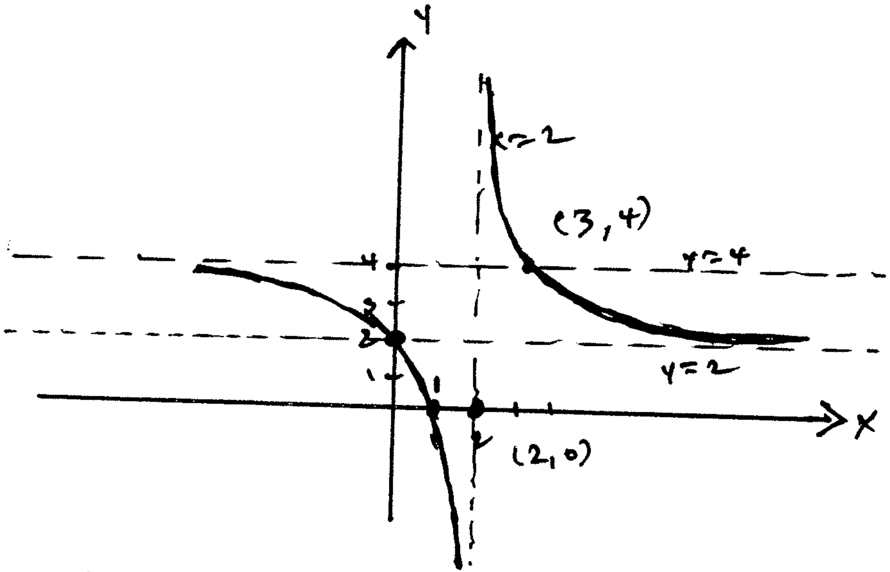
4

x	y
0	2
1	0
2	0
3	4

$x=2$ is an <u>odd</u> vert. asymp.

$y=4$ and $y=2$ are horizontal asymptotes.

4. (cont'd).



5 $\underline{x=-2}$ $f(-2)=\frac{8-(-8)}{4}=4$.

$\lim\limits_{x\to-2^-} f(x)=\lim\limits_{x\to-2^-}\frac{8-x^3}{4}=4$. $\lim\limits_{x\to-2^+} f(x)=\lim\limits_{x\to-2^+} 2-x=4$.

$\lim\limits_{x\to-2} f(x)=4=f(-2)$. Continuous at $x=-2$.

$\underline{x=2}$ $f(2)=2-2=0$. $\lim\limits_{x\to2^-} f(x)=\lim\limits_{x\to2^-} 2-x=0$,

$\lim\limits_{x\to2^+} f(x)=\lim\limits_{x\to2^+}\cos(x-2)=\cos 0=1$. $0\neq1$.

$\lim\limits_{x\to2} f(x)$ does not exist. The discontinuity is a <u>jump</u>.

6. $f(x)=\frac{x+1}{(x-3)(x+1)}$. Discontinuities at $x=3$ and $x=-1$.

$\lim\limits_{x\to-1} f(x)=\lim\limits_{x\to-1}\frac{1}{x-3}=-\frac{1}{4}$. Removable discontinuity at $x=-1$.

At $x=3$, $\lim\limits_{x\to3^+} f(x)=+\infty$, $\lim\limits_{x\to3^-} f(x)=-\infty$,

$f \xrightarrow[3]{-\quad|\quad+}$. Infinite discontinuity at $x=3$.