Math 1000 Solutions to wkst #2. F09.

1.(a) $\lim_{x\to\frac{1}{2}^+} g(x) = \lim_{x\to\frac{1}{2}^+} \cos(\pi x) = \cos\frac{\pi}{2} = 0$

$\lim_{x\to\frac{1}{2}^-} g(x) = \lim_{x\to\frac{1}{2}^-} (2x^3 - x^2) = \frac{2}{8} - \frac{1}{4} = 0$

$\lim_{x\to\frac{1}{2}} g(x) = 0$. (2.4.1)

(b) $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (2-x) = 3$

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} x = -1$

$\lim_{x\to -1} f(x)$ does not exist. (Thm 2.4.1).

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x = 1$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x-1)^2 = 0$

$\lim_{x\to 1} f(x)$ does not exist.

2. $\lim_{x\to -1^+} g(x) = \lim_{x\to -1^+} (c^2 x - 3) = -c^2 - 3$

$\lim_{x\to -1^-} g(x) = \lim_{x\to -1^-} (c^3 x + 1) = -c^3 + 1$

$-c^2 - 3 = -c^3 + 1$ if and only if $c^3 - c^2 - 4 = 0$

$c = 2$ is one root by inspection.

$$\begin{array}{r|rrrr} 2 & 1 & -1 & +0 & -4 \\ & & +2 & +2 & +4 \\ \hline & 1 & +1 & +2 & 0 \end{array}$$

$\therefore\ c^3 - c^2 - 4 = (c-2)(c^2 + c + 2)$.

$c = 2$ is the only <u>real</u> root. (Discriminant of $c^2 + c + 2 < 0$)

Only possible value for c is $c = 2$.

3. (a) $\displaystyle\lim_{x\to c} \frac{f(x)+g(x)}{2} = \frac{\lim_{x\to c}(f(x)+g(x))}{\lim_{x\to c} 2}$ T2, R4

$\displaystyle= \frac{\lim_{x\to c} f(x) + \lim_{x\to c} g(x)}{2}$ T2, R2

$\displaystyle= \frac{-3+4}{2} = \frac{1}{2}.$

(b) $\displaystyle\lim_{x\to c} f(x)\cdot g(x) = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$ T2, R3

$= -3\cdot 4 = -12.$

(c) $\displaystyle\lim_{x\to c} f(x)\sqrt{g(x)} = \lim_{x\to c} f(x) \cdot \lim_{x\to c}\sqrt{g(x)}$ T2, R3

$\displaystyle= -3\cdot\sqrt{\lim_{x\to c} g(x)}$ T2, R5

$= -4\cdot\sqrt{4} = -8.$

(d) $\displaystyle\lim_{x\to c} \frac{3f(x)}{1+2g(x)} = \frac{\lim_{x\to c} 3f(x)}{\lim_{x\to c}(1+2g(x))}$ T2, R4

$\displaystyle= \frac{3\lim_{x\to c} f(x)}{\lim_{x\to c} 1 + 2\lim_{x\to c} g(x)}$ T2, R1 (twice) T2, R2

$\displaystyle= \frac{-9}{1+8} = -1.$

4a $\displaystyle\lim_{x\to -3} \frac{x^2+5x+6}{12+x-x^2} = \lim_{x\to -3} \frac{(x+3)(x+2)}{(3+x)(4-x)} = \lim_{x\to -3}\frac{x+2}{4-x} = -\frac{1}{7}.$

4b $\displaystyle\lim_{x\to 2} \frac{3x-6}{\sqrt{x-1}-1} = \lim_{x\to 2} \frac{3(x-2)}{(\sqrt{x-1}-1)} \frac{(\sqrt{x-1}+1)}{(\sqrt{x-1}+1)}$

$\displaystyle= \lim_{x\to 2} \frac{3(x-2)(\sqrt{x-1}+1)}{(x-1-1)} = \lim_{x\to 2} 3(\sqrt{x-1}+1) = 6.$

4c $$\lim_{x\to-\sqrt{2}} \frac{x^4-x^2-2}{x^2-2} = \lim_{x\to-\sqrt{2}} \frac{(x^2-2)(x^2+1)}{x^2-2} = \lim_{x\to-\sqrt{2}} (x^2+1) = 3$$

4d $$\lim_{t\to 0} \frac{(2+t)^3-8}{t} = \lim_{t\to 0} \frac{(2+t)^3-2^3}{t}$$

$$= \lim_{t\to 0} \frac{(2+t-2)\left((2+t)^2+(2+t)\cdot 2+4\right)}{t} = \lim_{t\to 0}\left((2+t)^2+2(2+t)+4\right)$$

$$= 12 .$$

4e $$\lim_{x\to 2} \frac{\frac{2x-1}{x-1}-3}{x-2} = \lim_{x\to 2} \frac{\frac{2x-1-3(x-1)}{x-1}}{x-2}$$

$$= \lim_{x\to 2} \frac{2x-1-3x+3}{(x-1)(x-2)} = \lim_{x\to 2} \frac{-1\cancel{(2-x)}}{(x-1)\cancel{(x-2)}} = -1 .$$

4f $$\lim_{x\to 1} \left\{\frac{1}{x-1} - \frac{2}{x^2-1}\right\} = \lim_{x\to 1} \frac{(x+1)-2}{x^2-1}$$

$$= \lim_{x\to 1} \frac{x-1}{x^2-1} = \lim_{x\to 1} \frac{1}{x+1} = \frac{1}{2} .$$

4g $$\lim_{x\to 0} \frac{1-\cos x}{x\sin x} = \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x\sin x\,(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{(1-\cos^2 x)}{x\sin x(1+\cos x)} = \lim_{x\to 0} \frac{\sin^2 x}{x\sin x\,(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{\sin x}{x}\cdot\frac{1}{1+\cos x} = \lim_{x\to 0} \frac{\sin x}{x}\cdot \lim_{x\to 0} \frac{1}{1+\cos x}$$

$$= 1\cdot\frac{1}{2} = \frac{1}{2} .$$