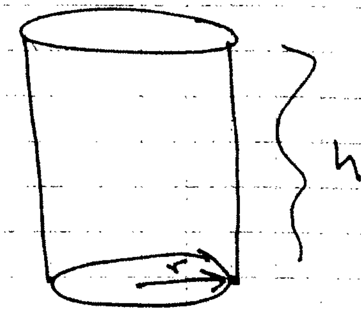
M1000 Solutions to Worksheet #10 F09

1. Let C represent the cost to be minimized.



$$C = C_{TB} + C_S$$

$$= d \cdot (2\pi r^2) + d(2\pi r h)$$

Constraint

$$\pi r^2 h = V$$

$$r > 0$$

$$C(r) = 2\pi d r^2 + 2\pi d r\left(\frac{V}{\pi r^2}\right)$$

$$= 2\pi d r^2 + \frac{2dV}{r}.$$

$$C'(r) = 4\pi d r - \frac{2dV}{r^2} = \frac{4\pi d r^3 - 2dV}{r^2}$$

Critical number is $r = \left(\frac{2dV}{4\pi d}\right)^{1/3} = \left(\frac{V}{2\pi}\right)^{1/3}$.

C' $-$ | $+$ $\longrightarrow r$. Minimum Cost when $r = \left(\frac{V}{2\pi}\right)^{1/3}$.

2a $\displaystyle\int \frac{3x^4 - 5}{4x^5}\,dx = \int \frac{3x^4}{4x^5} - \frac{5}{4x^5}\,dx = \int \frac{3}{4x} - \frac{5}{4x^5}\,dx$

$\displaystyle = \frac{3}{4}\int \frac{1}{x}\,dx - \frac{5}{4}\int x^{-5}\,dx = \frac{3}{4}\ln|x| + \frac{5}{16}x^{-4} + C$

b $\displaystyle\int \frac{1}{e^{4x}} - \sin(3-2x)\,dx = \int e^{-4x} - \sin(3-2x)\,dx$

$\displaystyle = \frac{e^{-4x}}{-4} + \frac{\cos(3-2x)}{-2} + C = -\frac{1}{4e^{4x}} - \frac{1}{2}\cos(3-2x) + C$

c $\displaystyle\int \frac{12}{\sqrt{1-4t}}\,dt = \int 12(1-4t)^{-1/2}\,dt = 12\int (1-4t)^{-1/2}\,dt$

$\displaystyle = 12\,\frac{(1-4t)^{1/2}}{1/2 \cdot (-4)} + C = -6(1-4t)^{1/2} + C$

2d $\displaystyle\int \frac{12}{1-4t}\,dt = 12\int \frac{1}{1-4t}\,dt = 12\,\frac{\ln|1-4t|}{-4} + C$

$= -3\ln|1-4t| + C$

2e $\displaystyle\int t^2(1-t^4)\,dt = \int t^2 - t^6\,dt = \frac{1}{3}t^3 - \frac{1}{7}t^7 + C$

2f $\displaystyle\int \frac{1}{(5+4x)^3}\,dx = \int (5+4x)^{-3}\,dx = \frac{(5+4x)^{-2}}{(-2)\cdot(4)} + C$

$\displaystyle = -\frac{1}{8}(5+4x)^{-2} + C = -\frac{1}{8(5+4x)^2} + C.$

2g $\displaystyle\int \csc\theta(\csc\theta + \cot\theta)\,d\theta = \int \csc^2\theta + \csc\theta\cot\theta\,d\theta$

$= -\cot\theta - \csc\theta + C.$

3 $\displaystyle f'(x) = \int f''(x)\,dx = \int 3e^{2x} + \sin(2x)\,dx$

$\displaystyle = \frac{3}{2}e^{2x} - \frac{1}{2}\cos(2x) + C.$ If $f'(0) = 2$

then $\displaystyle 2 = \frac{3}{2} - \frac{1}{2} + C$ so $C = 1$.

$\displaystyle f'(x) = \frac{3}{2}e^{2x} - \frac{1}{2}\cos(2x) + 1.$

$\displaystyle f(x) = \int f'(x)\,dx = \int \frac{3}{2}e^{2x} - \frac{1}{2}\cos(2x) + 1\,dx$

$\displaystyle = \frac{3}{4}e^{2x} - \frac{1}{4}\sin(2x) + x + D.$ When $f(0) = 1$,

$\displaystyle 1 = \frac{3}{4} - 0 + 0 + D.$ $\therefore$ $\displaystyle D = \frac{1}{4}.$

$\displaystyle f(x) = \frac{3}{4}e^{2x} - \frac{1}{4}\sin(2x) + x + \frac{1}{4}.$

4a $\displaystyle\int_{-2}^{3}(5+x-6x^2)\,dx = \left(5x+\frac{1}{2}x^2-2x^3\right)\Big|_{-2}^{3}$

$= (15+\frac{9}{2}-54)-(-10+2+16) = -47+\frac{9}{2} = -\frac{85}{2}$

b $\displaystyle\int_{-6}^{-1} 8\,dx = 8x\Big|_{-6}^{-1} = (-8)-(-48) = 40.$

c $\displaystyle\int_{1}^{4}\sqrt{16t^5}\,dt = 4\int_1^4 t^{5/2}\,dt = \frac{8}{7}t^{7/2}\Big|_1^4 = \frac{8}{7}\left(4^{7/2}-1^{7/2}\right) = \frac{8(127)}{7} = \frac{10}{\ }$

d $\displaystyle\int_{-2}^{-1}\frac{(4-3x)^2}{x^3}\,dx = \int_{-2}^{-1}\frac{16-24x+9x^2}{x^3}\,dx = \int_{-2}^{-1}\frac{16}{x^3}-\frac{24}{x^2}+\frac{9}{x}\,dx$

$\displaystyle= \int_{-2}^{-1} 16x^{-3}-24x^{-2}+9x^{-1}\,dx = \left(-8x^{-2}+24x^{-1}+9\ln|x|\right)\Big|_{-2}^{-1}$

$= (-8-24+0)-(-2-12+9\ln 2) = -18-9\ln 2$

e $\displaystyle\int_0^3\frac{8}{4x+3}\,dx = 2\ln|4x+3|\Big|_0^3 = 2(\ln 15-\ln 3) = 2\ln 5$

$= \ln 25.$

f $\displaystyle\int_0^{\ln 2}\frac{e^{4x}-4}{e^{2x}}\,dx = \int_0^{\ln 2} e^{4x}-4e^{-2x}\,dx = \left(\frac{e^{4x}}{4}+2e^{-2x}\right)\Big|_0^{\ln 2}$

$= \left(\frac{e^{4\ln 2}}{4}+2e^{-2\ln 2}\right)-\left(\frac{1}{4}+2\right) = (4+\frac{1}{2})-(\frac{1}{4}+2) = 2\frac{1}{4} = \frac{9}{4}$

g $\displaystyle\int_0^1\frac{u^2-u-2}{u-2}\,du = \int_0^1\frac{(u-2)(u+1)}{u-2}\,du = \int_0^1 u+1\,du$

$= (\frac{1}{2}u^2+u)\Big|_0^1 = (\frac{1}{2}+1)-0 = 3/2.$

h $\displaystyle\int_0^{\pi/2}\sin\left(\frac{3x}{2}\right)dx = -\frac{2}{3}\cos\left(\frac{3x}{2}\right)\Big|_0^{\pi/2} = -\frac{2}{3}\left\{\cos\frac{3\pi}{4}-\cos 0\right\}$

$= -\frac{2}{3}\left(-\frac{\sqrt{2}}{2}-1\right) = \frac{2}{3}\,\frac{\sqrt{2}+2}{2} = \frac{\sqrt{2}+2}{3}.$