MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAMINATION | **Mathematics 1000** | **WINTER 2007**

Marks

1. Evaluate the following limits, showing your work. Assign ∞ or $-\infty$ as appropriate.

[3] a) $\lim_{x\to 2} \dfrac{3-\sqrt{x+7}}{x-2}$ [3] b) $\lim_{x\to 0} \dfrac{\sin(6x)}{x\cos^2 x}$ [3] c) $\lim_{x\to -\infty} \dfrac{3x+1}{\sqrt{4x^2-5}}$

[3] d) $\lim_{x\to -1^-} \dfrac{2x^2+x-1}{x^2+2x+1}$ [3] e) $\lim_{x\to 2^-} \dfrac{|x^2-4|}{x-2}$

[6] 2. Use the DEFINITION OF DERIVATIVE to find $f'(x)$ for $f(x) = \sqrt{x+2}$.

[4] 3. Let $f(x) = \begin{cases} x^2+1, & x<1 \\ 2x, & x=1 \\ 3x^2-1, & x>1 \end{cases}$

Use the definition of continuity to determine if $f(x)$ is continuous at $x = 1$.

[16] 4. Differentiate each function and make any appropriate simplifications.

a) $y = \dfrac{1+\sin x}{1+\cos x}$ b) $f(x) = \tan^2(3x^4-5)$ c) $y = \cos(\ln x) + \ln(x\cos x)$

d) $y = \dfrac{5^x(\sin x)^4}{\sqrt{3x^5-7x}}$

[5] 5. Find the equation of the tangent line to the curve $f(x) = (x-1)e^x$ at the point $(1,0)$.

[8] 6. A manufacturer wishes to design an open box having a square base and a surface area of 108 square metres. What dimensions will give a box with maximum volume?

[6] 7. Find y' and y'' for $xy + x - 2y = 1$.

[8] 8. A plane flies directly over a man and is 6 kilometres above his head. The distance from the plane to the man is increasing at the rate of 400 kilometres per hour when the distance from the plane to the man is 10 kilometres. How fast is the plane moving?

[8] 9. Sketch the graph of a function $f(x)$ that satisfies all of the following conditions. Label any asymptotes, extrema, and inflection points.

Domain of $f = \{x \mid x \neq -1\}$. $f(0) = 1, f(1) = 2, f(2) = 4, f(3) = 1, f(-2) = 0$

$f'(x) > 0$ on $(0,2)$, $f'(x) < 0$ on $(-\infty,-1)$, $(-1,0)$ and $(2,\infty)$, and $f'(x) = 0$ at $x = 0$ and $x = 2$

$f''(x) > 0$ on $(-1,1)$ and $(3,\infty)$, $f''(x) < 0$ on $(-\infty,-1)$ and $(1,3)$, and $f''(x) = 0$ at $x = 1$ and $x = 3$

$\lim_{x\to -\infty} f(x) = 3$, $\lim_{x\to -1^-} f(x) = -\infty$, $\lim_{x\to -1^+} f(x) = +\infty$ and $\lim_{x\to\infty} f(x) = 0$

[7] 10. Given that $f'(x) = \dfrac{-10x}{(x^2-4)^2}$ and $f''(x) = \dfrac{10(3x^2+4)}{(x^2-4)^3}$,

a) Use calculus to find all intervals on which $f(x)$ is increasing or decreasing and use the first derivative test to give the x-coordinate of each relative maximum and relative minimum, if they exist.

b) Use calculus to find all intervals on which $f(x)$ is concave up or concave down and find the x-coordinate of each point of inflection, if they exist.

11. Find each the following integrals.

[3] a) $\displaystyle\int \left(\sin(3x-1) + \frac{1}{x}\right) dx$ [3] b) $\displaystyle\int_{-1}^{1} (3x^2+1)\, dx$

[6] 12. Find the area of the region bounded by the graphs of $y = x^2 + 2$ and $y = x + 4$.

[5] 13. Answer **ONE** of the following.

a) Use the DEFINITION OF DERIVATIVE to find $f'(x)$ if $f(x) = \dfrac{1}{g(x)}$.

b) Find a function $f(x)$ such that the point $(-1, 1)$ is on the graph of $y = f(x)$, the slope of the tangent line at $(-1, 1)$ is 2 and $f''(x) = 6x + 4$.

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[100]

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAMINATION | **Mathematics 1000** | **WINTER 2003**

Marks

1. Evaluate each limit or explain why it does not exist. Assign ∞ or $-\infty$ when appropriate.

[3] a) $\lim\limits_{x \to 5} \dfrac{25 - x^2}{x^2 - 2x - 15}$ [3] b) $\lim\limits_{x \to -1} \dfrac{3x}{(x+1)^2}$ [3] c) $\lim\limits_{x \to 0} \dfrac{\sin x}{1 - \sqrt{1+x}}$

[4] 2. a) Use the definition of derivative to find $f'(x)$ given $f(x) = \sqrt{2x+1}$.

[4] b) Find the equation of the tangent line to $f(x) = \sqrt{2x+1}$ at $x = 1$.

3. Let $f(x) = \begin{cases} \dfrac{x^2 - 4}{x - 2}, & x \geq 0 \\ \dfrac{4}{x+2}, & x < 0 \end{cases}$.

[4] a) Use the definition of continuity at a point to show that f is continuous at $x = 0$.

[4] b) Determine any discontinuities of f and show whether each discontinuity is removable or non-removable.

[8] 4. An airplane flying at an altitude of 10 km passed directly over a radar antenna. When the distance from the radar antenna to the plane is 26 km, the radar detected that the distance to the plane was changing at a rate of 200 km per hour. What was the speed of the plane?

[20] 5. Find the derivative $\frac{dy}{dx}$. Show all work and simplify your answer where possible.

a) $y = \dfrac{x^2 + c^2}{x^2 - c^2}$ (c is constant) b) $y = e^{x^2}(x^2 + 1)$ c) $y - x = \tan y$

d) $y = \dfrac{(x^2+1)^x}{x}$ e) $y = \sin\sqrt{x} + \sqrt{\sin x}$

[6] 6. The velocity of a particle along a straight line is given by $v(t) = t^2 - 2t + 3$, where the position is measured in centimetres and the time t in seconds. If $s(0) = 1$, find the position of the particle at time $t = 1$ seconds.

[7] 7. Determine the largest possible area for a right triangle whose hypotenuse is 6 cm.

[12] 8. Given $f(x) = \dfrac{2x^2 + x + 6}{x^2 + 3}$

a) Find any vertical and horizontal asymptotes to the graph of f.

b) Given that $f'(x) = \dfrac{3 - x^2}{(x^2+3)^2}$, find any intervals on which f is increasing or decreasing.

c) Given that $f''(x) = \dfrac{2x(x^2 - 9)}{(x^2+3)^3}$, find any intervals on which the graph of f is concave up or concave down.

d) Find the exact coordinates of any relative extreme points and inflection points on the graph of f.

[9] 9. Find each of the following:

a) $\int (\tan^2 x + 1)\, dx$ b) $\int \frac{3}{5-2u}\, du$ c) $\int_{-3}^{1} (x+3)^{-\frac{1}{2}}\, dx$

[7] 10. Sketch the graph of the function f that satisfies all of the given conditions:

$f(-3) = f(3) = 0$ and $f(0) = -27$
$f'(-3) = f'(1) = 0$
$f'(x) < 0$ if $-3 < x < 1$
$f'(x) > 0$ if $x < -3$ and $x > 1$
$f''(x) < 0$ if $x < -1$ and $f''(x) > 0$ if $x > -1$
$f''(-1) = 0$

[6] 11. Answer **ONE** of the following:

a) Find the area of the region in the xy-plane bounded by the graphs of $y = 4 - x^2$ and $y = x^2 + 6x + 4$.

b) Find the equation of the parabola $y = ax^2 + bx + c$ that passes through (0, 1) and is tangent to the line $y = x - 1$ at (1, 0).