MATH 1000, Slot F05   Worksheet 9

Due: Nov. 24/2009 (Tuesday) by 5pm

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1. Find the intervals of concavity and any points of inflection of the following functions. Show all work and include the "sign pattern" of the second derivative.

   (a) $f(x) = x^3 - 6x^2 + 9x - 1$

   (b) $f(x) = x - 2\sin x$. Just consider the interval $[-3\pi, 3\pi]$.

   (c) $f(x) = x^4 - 8x^2$

   (d) $f(x) = x^{\frac{2}{3}}(5 - 2x)$

   (e) $f(x) = 2xe^{-2x}$

2. Sketch the graph of each function showing all relative extrema, inflection points, intercepts and asymptotes. SHOW ALL WORK.

   (a) $f(x) = 3x^5 + 5x^4$
   (b) $f(x) = x^2/(x - 3)$

3. Sketch the graph of a function f, with domain $[-3, \infty)$ and continuous except at $x = 0$, satisfying all of the following.

   (a) $f(-1) = f(0) = f(2) = -1$, $f(-3) = 4$
   (b) $f'(-1) = f'(2) = 0$
   (c) $f'(x) > 0$ on the intervals $(-1, 0)$ and $(2, \infty)$ while $f'(x) < 0$ on $(-3, -1)$ and $(0, 2)$.
   (d) $f''(x) > 0$ on the intervals $(-3, 0)$ and $(0, 5)$ while $f''(x) < 0$ on $(5, \infty)$.
   (e) $\lim_{x \to \infty} f(x) = 1$ and $\lim_{x \to 0} f(x) = \infty$.

4. Find any absolute extrema of each of the following. For (a), (c) and (d) justify your answers with the sign pattern of the derivative and monotonicity. For (b) just sketch the graph by hand.

   (a) $f(x) = xe^x$ on domain $[-3, \infty)$.
   (b) $f(x) = \begin{cases} x^2 & \text{if } -1 \le x < 0 \\ 2 - x^2 & \text{if } 0 \le x \le 1 \end{cases}$

   (c) $f(x) = 2x^5 - 10x + 5$ on $\left[-\frac{3}{2}, \frac{3}{2}\right]$

   (d) $f(x) = \frac{1}{1 + x^2}$ over $\mathbb{R}$.

5. An open box having a square base is to be made from 108 square cm. of material. Find the dimensions of the box that will give it maximum volume.

6. From a square piece of cardboard, 30 centimetres on each side, an open-topped box is to be constructed by cutting squares from the corners and turning up the sides. What are the dimensions of the box of largest volume?