MATH 1000, Slot F05 — Worksheet 8

1. A baseball is thrown vertically upward from field level directly below the highest point in the Rogers Centre in Toronto. Assume the ball left the player's hand 6 feet above the playing field at a velocity of 90 miles per hour. If the Rogers Centre is 310 feet high at that point, will the baseball hit the retractable roof? If not, how close will it come?

2. The rectilinear motion of an object is given by the displacement equation $s = t^3 - 4t^2 + 4t$ where s is in centimetres and t is in seconds.

   (a) Find $v(t)$ and $a(t)$.

   (b) When is the object at rest?

   (c) When is the object moving right and when it is moving left? Assume $t \geq 0$.

   (d) What is the acceleration of the object when it is at rest?

   (e) What is the total *distance* travelled in the first four seconds?

3. Show that the displacement equation given by

$$s(t) = 4\sin(kt + c) - 3\cos(kt + d)$$

   represents simple harmonic motion.

4. Find all critical numbers, intervals of monotonicity and identify any local (or relative) extrema of the following functions. Show all work and include the "sign pattern" of the derivative.

   (a) $f(x) = x^3 - 6x^2 + 9x - 1$

   (b) $f(x) = \dfrac{x^2 - 3}{3x^2 - 1}$

   (c) $y = \cos x - \sin x$ in the interval $0 \leq x \leq 2\pi$.

   (d) $f(x) = x^3(x + 2)^2$

   (e) $f(x) = x^{\frac{2}{3}}(5 - 2x)$

   (f) $f(x) = 2xe^{-2x}$