MATH 1000, Slot F05 Worksheet 7

Due: Nov. 3/2009 (Tuesday) by 5pm

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1. A baseball diamond has the shape of a square with sides 90 feet long. A player is running from second to third at a speed of 28 feet per second. At what rate is the player's direct distance from home plate changing at the instant she is 30 feet from third?

2. A conical tank (with vertex down) is 10 feet across at the top and 12 feet deep. If water is flowing into the tank at a rate of 10 cubic feet per minute, find the rate of change of the depth of water the instant it is 8 feet deep.

3. An airplane flies at an altitude of 5 km toward a point directly above an observer at ground level. The speed of the plane is 600 km per hour. Find the rate at which the angle, θ, between the ground and the observer's line of sight to the airplane is changing at the instant when the angle is 45 degrees.

4. A kite is flying in a horizontal wind at a constant height of 300 feet. When 500 feet of string are out, the kite is pulling the string out at a rate of 20 feet per second. What is the wind velocity?

5. The length of a rectangle is increasing at the rate of 2 metres per second and the width is decreasing at the rate of 1 metre per second. When the length is 4 metres and the width is 1.5 metres, find the rate of change of (a) the area (b) the perimeter and (c) the length of the diagonal.

6. Combine to a single logarithm: $3\log_b x - \dfrac{1}{2}\log_b(1+x) + \log_b(1+x^2)$

7. Expand: $\log_b\left(2^x \tan x\right)$

8. Solve for x:

   (a) $\log_5(3x+1) + \log_5(x+1) = 1$

   (b) $4^{x-2} = (2^x)^3$

9. Find the derivative of each of the following functions.

   (a) $f(x) = \ln\left(\sin^2 x\sqrt{(1+2x^3)^3}\right)$ (b) $f(x) = \ln\left(\dfrac{(1-x)^3(1-3x)}{(x^2+2)^5}\right)$

10. Use **logarithmic differentiation** to find the derivative, y', of each function.

    (a) $y = \dfrac{x(x-1)(x-2)}{(x-3)^3}$ (b) $y = \dfrac{\sin^2 x \tan^4 x}{(x^2+1)^2}$

    (c) $y = x^{\cos x}$ (d) $y = (\ln x)^{x^2}$