MATH 1000, Slot F05 Worksheet 4

Due: Oct. 6/2009 (Tuesday) by 5pm

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1. Let $f(x) = \begin{cases} \sqrt{2-x} & \text{if} \quad x \leq 1 \\ \frac{x^3+1}{3-x} & \text{if} \quad x > 1. \end{cases}$

   Determine, giving a very brief reason for each, whether or not $f(x)$ is continuous on each of the open intervals $(-\infty, 1)$, $(-\infty, 2)$, $(-\infty, 3)$ and $(-\infty, 4)$.

2. Briefly explain why $f(x) = (x-1)^2 - \ln x - 1$ has an absolute maximum, an absolute minimum and a root in the interval $[1, e]$. Find exactly, or approximate using your calculator if necessary, the x-coordinates of these points.

3. For each function, set up and then evaluate $\lim\limits_{w \to x} \dfrac{f(w) - f(x)}{w - x}$ or $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ (your choice). In other words, find $f'(x)$ in each case.

   (a) $f(x) = x^2 - 5x + 1$.

   (b) $f(x) = \frac{x-1}{1-2x}$.

   (c) $f(x) = \sqrt{3x-2}$.

4. If the derivative of $f(x)$ is $f'(x) = 2 - 7x - 15x^2$, find each of the following:

   (a) the slope of the tangent line and the slope of the normal line to $f(x)$ at $x = 1$,

   (b) the values of x where the graph of $f(x)$ has a horizontal tangent line.

5. Sketch the graph of $y = \sin x$ on a sheet of paper or on your calculator. By inspecting the graph, what should the equation of the tangent line be at the point on the graph with x-coordinate $x = \frac{\pi}{2}$.

6. Find the derivative of each function using the rules for derivatives on pages 66 and 67 of the Course Notes. Indicate the Table 2 Rule that you used. Then, find the slope of the tangent line to the curve at $x = -1$.

   (a) $f(x) = 3x^3 - 2x^2 + 4x - 3$

   (b) $f(x) = \frac{1}{x^2} - e^x$

   (c) $f(x) = \frac{1+4x-3x^2}{5}$

7. For what two values of x does $g(x) = 2x^3 - x^2 - 8x + 33$ have a horizontal tangent? Give the equation of one of the horizontal tangent lines (your choice).