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Memorial University of Newfoundland

NOV 9/09

$\begin{cases} 4.4 \\ 4.6 \end{cases}$

LOCAL EXTREMA

CRITICAL NUMBER, c

CRITICAL POINT $(c, f(c))$

$\uparrow$

C.N.

1ST DERIVATIVE TEST FOR LOCAL EXTREMA

GIVEN SUPPOSE $x = c$ IS A CRITICAL NUMBER AND $f(x)$ IS CONTINUOUS AT c. [$(c, f(c))$ IS A C.P.]

THEN (i) $f' \xrightarrow[c]{+ \quad | \quad -} x$ LOCAL MAX

(ii) $f' \xrightarrow[c]{- \quad | \quad +} x$ LOCAL MIN

(iii) OTHERWISE, NEITHER.

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EXAMPLES

1) $f(x) = x^2 (x+2)^5$

Sol'n $f'(x) = x^2\, 5(x+2)^4 + (x+2)^5\, 2x$

$= x(x+2)^4 \left(5x + 2(x+2)\right)$

$= x(x+2)^4 (7x+4)$ (1)

f'	$+$	-2	$+$	$-4/7$	$-$	0	$+$	x

(2)

NO EXTREMUM AT $x = -2$
LOCAL MAX AT $x = -4/7$
LOCAL MIN AT $x = 0$

(3) 1ST DER. TEST.

2) $f(x) = x \ln x$ $D_f = (0, \infty)$.
$x > 0$.

Sol'n

$f'(x) = x \cdot \frac{1}{x} + \ln x$

$f'(x) = 1 + \ln x$

CRITICAL NUMBERS

FOR WHAT x, IF ANY, DOES $1 + \ln x = 0$?

$\ln x = -1$?

$e^{\ln x} = e^{-1}$ OR

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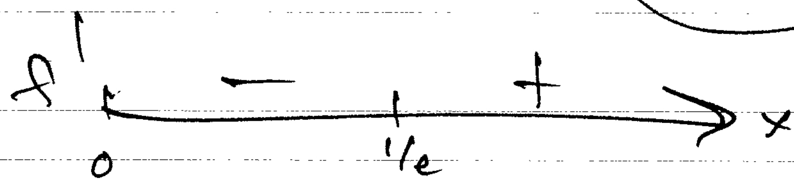
$x = e^{-1}$ or $x = \frac{1}{e}$. CRITICAL NUMBER



$e^2 > e$

$\frac{1}{e^2} < \frac{1}{e}$

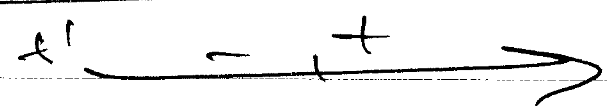
$$\boxed{\begin{aligned} & a > b > 0 \\ & \frac{1}{a} < \frac{1}{b} \end{aligned}}$$

$2e > e$

$\frac{1}{2e} < \frac{1}{e}$

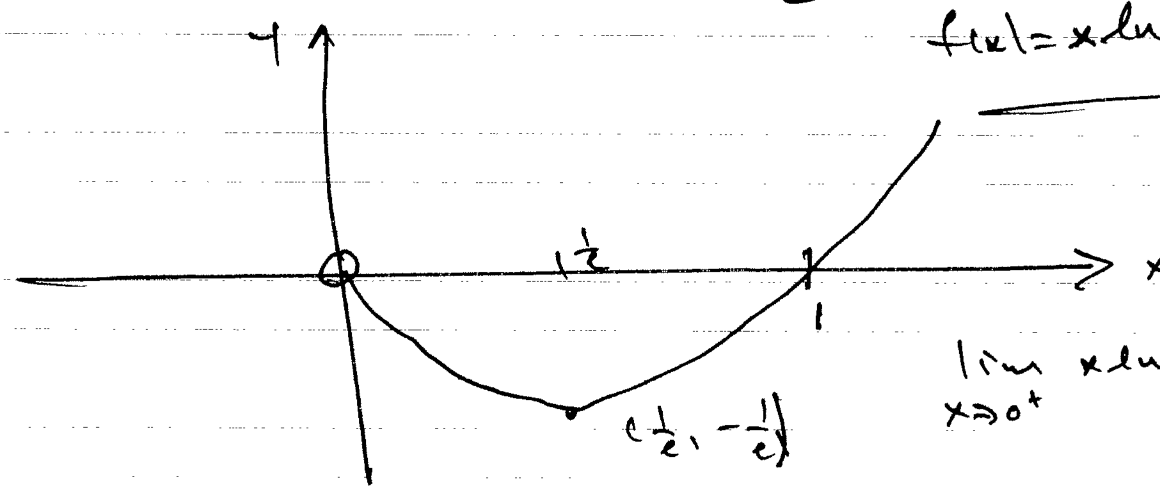
$$\boxed{\begin{aligned} f'(x) &= 1 + \ln x \\ f'\left(\tfrac{1}{e^2}\right) &= 1 + \ln\left(\tfrac{1}{e^2}\right) \\ &= 1 + \ln 1 - \ln e^2 \\ &= 1 + 0 - 2\ln e \\ &= 1 - 2 = -1 \end{aligned}}$$

AT $\left(\frac{1}{e}, -\frac{1}{e}\right)$



LOCAL MINIMUM AT $x = \frac{1}{e}$

$f(x) = x \ln x$



$\lim_{x \to 0^+} x \ln x = 0$

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$f(-x) = -f(x)$

(3) $f(x) = \dfrac{x}{1+x^2}$

SOL'N $f'(x) = \dfrac{(1+x^2)\,1 - x\,2x}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2}$

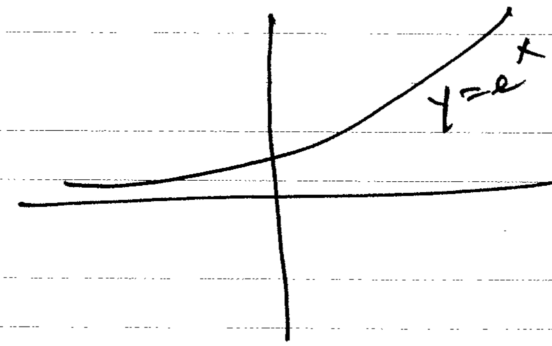
f': $-$ $\quad -1 \quad$ $+$ $\quad 1 \quad$ $-$ $\rightarrow x$

LOCAL MIN AT $x=-1$, $f(-1) = -\frac{1}{2}$

LOCAL MAX AT $x=1$ $f(1) = \frac{1}{2}$.

(4) $f(x) = x^{2/3} e^{-x}$

SOL'N $f(x) = \dfrac{x^{2/3}}{e^x}$



$D_f = \mathbb{R}$.

$f'(x) = x^{2/3}(-e^{-x}) + \frac{2}{3}x^{-1/3}e^{-x}$

$= \frac{1}{3}e^{-x}x^{-1/3}(-3x+2)$

$= \dfrac{2-3x}{3e^x x^{1/3}}$

$2-3x=0$

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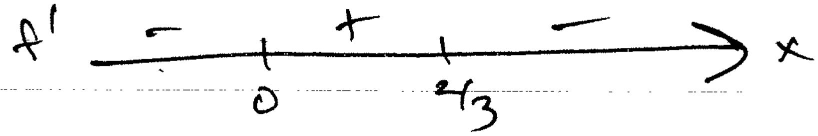

AT $x=0$, THERE IS A LOCAL MIN

AT $x=2/3$, THERE IS A LOCAL MAX

$(0,0)$, $\left(\frac{2}{3}, f\left(\frac{2}{3}\right)\right)$ ARE BOTH CRITICAL POINTS.