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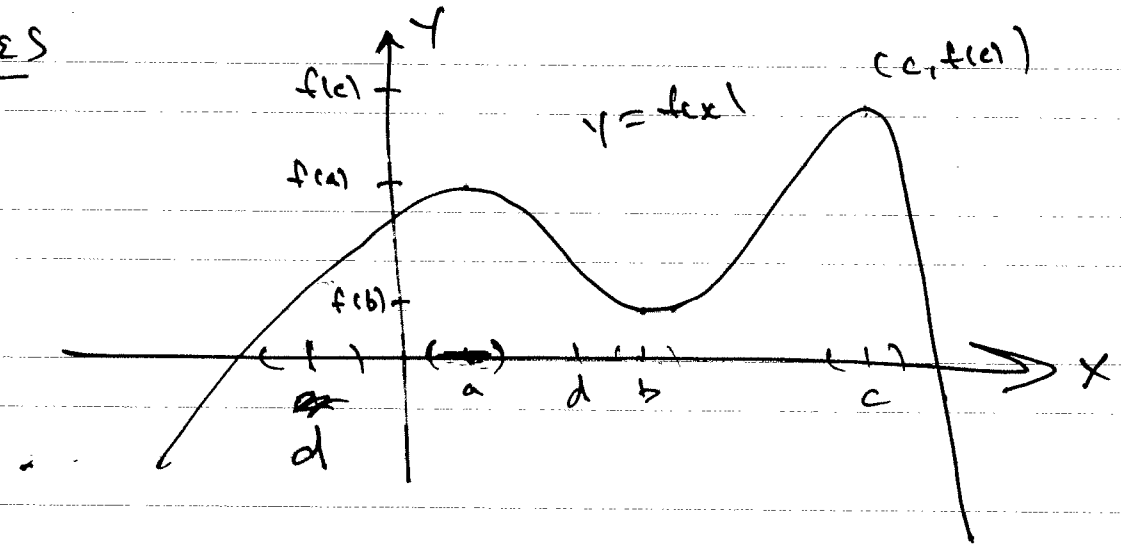
Nov 6/09.

§4.6 Absolute Extrema
Local Extrema

THERE ARE TWO KINDS OF EXTREME VALUES OF A FUNCTION, $f(x)$.
1) ABSOLUTE
2) LOCAL (OR RELATIVE)

EXAMPLES

1)



Local Maximum of $f(a)$. It occurs at $x=a$
Local Max of $f(c)$. It occurs at $x=c$
Local Minimum of $f(b)$. It occurs at $x=b$.

Absolute Maximum is $f(c)$. It occurs at $x=c$.

NO Absolute Minimum.

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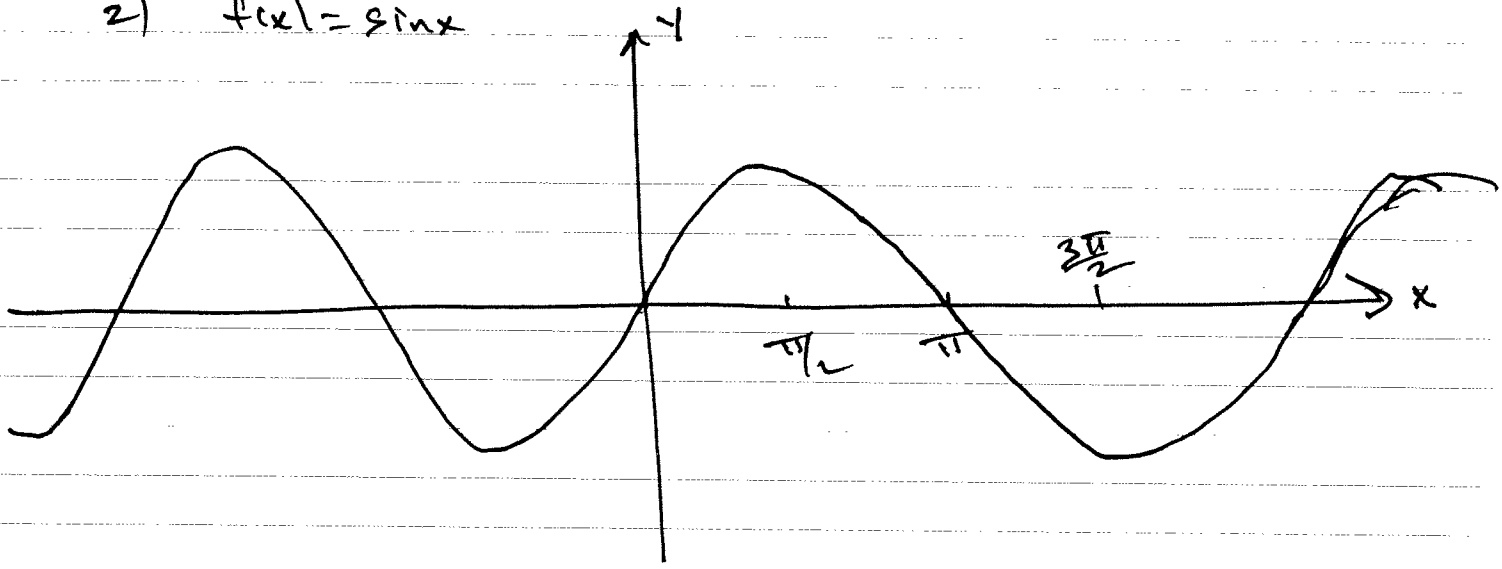
2) $f(x) = \sin x$



Local Maximum at $x = \pi/2$. Maximum is 1.

Local Maximum of 1 at $x = \pi/2 + 2k\pi$

$k = 0, \pm 1, \pm 2, \ldots$

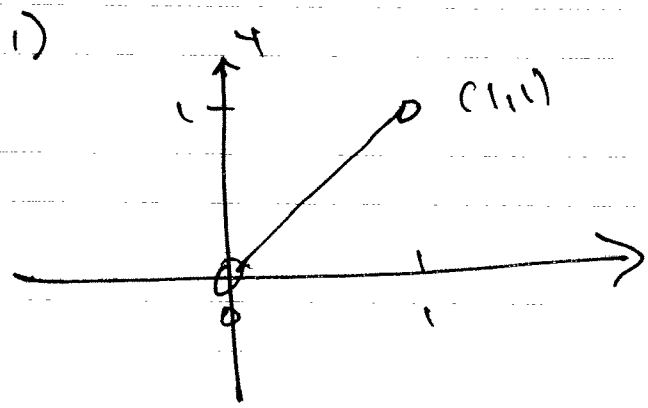
Local Minimum of -1 at $x = \frac{3\pi}{2} + 2k\pi$.

THE ABSOLUTE MAX IS $+1$.

THE ABS. MIN IS -1.

3) $f(x) = x$ $\quad D_f = (0,1)$

NO ABS. MAX
NO ABS. MIN



BUT, CHANGE D_f TO $[0,1)$

ABS. MIN. NO ABS MAX.

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RECALL EXTREME VALUE THEOREM.

DEFINITIONS (ABSOLUTE EXTREMA)

$f(c)$ IS THE **ABSOLUTE MAXIMUM** (**MINIMUM**) OF $f(x)$ WITH DOMAIN D_f IF $f(c) \geq f(x)$ ($f(c) \leq f(x)$) FOR EVERY x IN D_f.

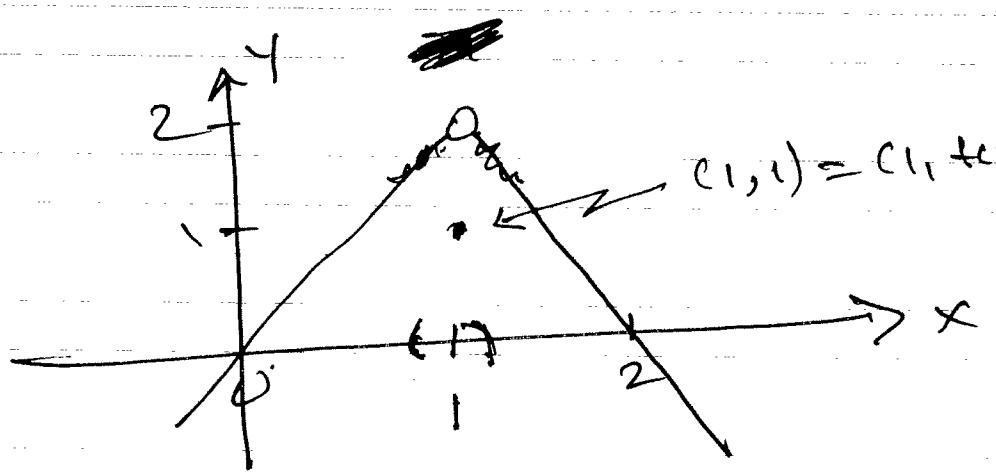
DEFINITIONS (LOCAL EXTREMA)

$f(c)$ IS A **LOCAL MAXIMUM** (**MINIMUM**) OF $f(x)$ HAVING D_f IF

(i) c IS IN SOME OPEN INTERVAL, I, IN D_f

AND (ii) $f(c)$ IS THE ABSOLUTE MAXIMUM (MINIMUM) OF $f(x)$ JUST ON I.

Ex 4



$f(1) \leq f(x)$? $f(1) \leq f(x)$ $\forall x \in I$

1 is a local MIN. It occurs at $x = 1$.

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(4)

LOCAL MAX $f(c) \geq f(x)$ FOR ALL x "NEAR" c

LOCAL MIN $f(c) \leq f(x)$ FOR ALL x "NEAR" c.

HOW TO FIND LOCAL EXTREMA (WITHOUT THE GRAPH).

FIRST DERIVATIVE TEST FOR LOCAL (RELATIVE) EXTREMA

GIVEN (1) $x = c$ IS A CRITICAL NUMBER OF $f(x)$
(2) $f(x)$ IS CONTINUOUS AT $x = c$.

((1) + (2) MEAN $(c, f(c))$ IS A CRITICAL POINT)

THEN (i) $f(c)$ IS A LOCAL MAX OF $f(x)$ IF

THE SIGN PATTERN OF f' IS: f' + | − $\longrightarrow x$ (| at c)

(ii) $f(c)$ IS A LOCAL MIN OF $f(x)$ IF

THE SIGN PATTERN OF $f'(x)$ IS: f' − | + $\longrightarrow x$ (| at c)

(iii) $f(c)$ IS NEITHER A MAX NOR A MIN

IF THE SIGN PATTERN OF f' IS EITHER

f' + | + $\longrightarrow x$ OR f' − | − $\longrightarrow x$ (| at c)

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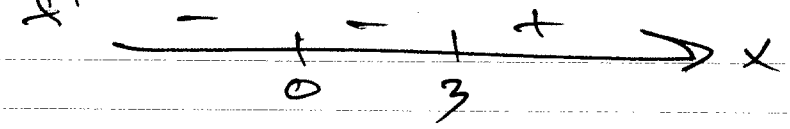
LOCATE AND IDENTIFY ANY LOCAL EXTREMA.

(1) $f(x) = x^4 - 4x^3$.

SOLN FIND f' AND FACTOR IT.

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$$

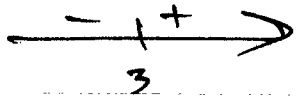
GET THE SIGN PATTERN OF f'.



USE 1ST DERIVATIVE TEST FOR LOCAL EXT

CHECK THE CRITICAL POINTS $(0, f(0))$ $(3, f(3))$.

$x=0$ NEITHER A MAX NOR A MIN AT $x=0$.

$x=3$ 

LOCAL MIN AT $x=3$.

LOCAL MIN IS $f(3) = 3^4 - 4 \cdot 3^3$

$= 3^3(3-4) = -27$.