Department of Mathematics and Statistics
Memorial University of Newfoundland

NOV 5/09.

§4.4, 4.6 MONOTONIC FUNCTIONS
LOCAL EXTREMA

MONOTONICITY RULE

IF $f'(x) > 0$ ON THE OPEN INTERVAL, I, THEN $f(x)$ IS INCREASING ON I

IF $f'(x) < 0$ ON THE INTERVAL I THEN $f(x)$ IS DECREASING ON I.

CRITICAL NUMBERS

THE NUMBER c IS A CRITICAL NUMBER OF $f(x)$ IF EITHER (i) $f'(c) = 0$
(ii) $f'(c)$ IS UNDEFINED.

CRITICAL POINT

THE POINT (ORDERED PAIR) $(c, f(c))$ IS A CRITICAL POINT OF $f(x)$ IF
(i) c IS A CRITICAL NUMBER
AND (ii) $f(x)$ IS CONTINUOUS AT c.

EX WHAT DOES $(2, 5)$ IS A C.P. OF $f(x)$ TELL YOU?

$x = 2$ IS A C.N.
$f(2) = 5$
$f(x)$ IS CONTINUOUS AT $x = 2$.

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FINDING INTERVALS OF MONOTONICITY (§ 4.4).

PROCEDURE GIVEN $y = f(x)$

(1) FIND $f'(x)$ AND FACTOR IT.

(2) GET THE SIGN PATTERN OF $f'(x)$

(3) USE THE MONOTONICITY RULE.

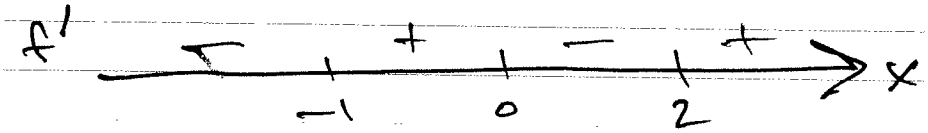
EXAMPLES: FIND THE INTERVALS OF MONOTONICITY

(1) $f(x) = 3x^4 - 4x^3 - 12x^2 - 1$

SOLN $f'(x) = 12x^3 - 12x^2 - 24x$

$= 12x(x^2 - x - 2)$

$= 12x(x+1)(x-2)$



$f(x)$ IS INCREASING ON $(-1, 0), (2, \infty)$

DECREASING ON $(-\infty, -1), (0, 2)$

CRITICAL POINTS: $(0, -1)$ $(-1, f(-1))$ $(2, f(2))$

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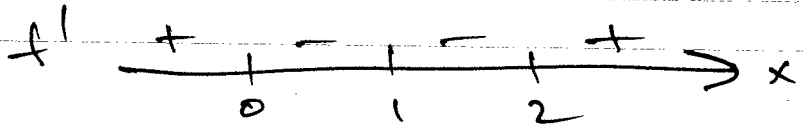
(2) $f(x) = \dfrac{x^2}{x-1}$

SOLN $f'(x) = \dfrac{(x-1)2x - x^2 \cdot 1}{(x-1)^2}$

$= \dfrac{2x^2 - 2x - x^2}{(x-1)^2} = \dfrac{x^2 - 2x}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2}$



$f(x)$ IS INCREASING ON $(-\infty, 0)$, $(2, \infty)$

$f(x)$ IS DECREASING ON $(0, 1)$, $(1, 2)$

(3) $f(x) = x^{2/5}(x-2)$

SOLN $f'(x) = x^{2/5} + (x-2)\frac{2}{5}x^{-3/5}$ $\quad n = \frac{2}{5}$,

$= \frac{1}{5}x^{-3/5}(5x + 2(x-2))$

$x^{-3/5} ? x^{2/5} = x$

$= \dfrac{7x - 4}{5x^{3/5}}$

$f(x) = x^{2/5}(x-2)$ IS INCREASING ON $(-\infty, 0)$, $(\frac{4}{7}, \infty)$

DECREASING ON $(0, 4/7)$.

$(0,0)$ $(\frac{4}{7}, f(4/7))$ BOTH CRITICAL POINTS.

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$D_f = [0, 2\pi]$

(4) $f(x) = e^x \sin x \qquad 0 \le x \le 2\pi$

SOLN $f'(x) = e^x \cos x + \sin x \cdot e^x$

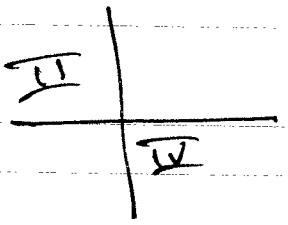
$f'(x) = e^x (\cos x + \sin x)$ $\qquad a \cdot b = 0$

? CAN $\cos x + \sin x = 0$?

$\cos x + \sin x = 0$ if and only if

$\sin x = -\cos x$

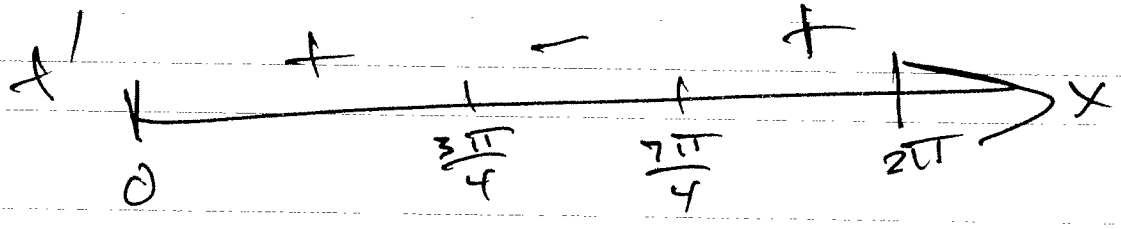
if $\dfrac{\sin x}{\cos x} = -1$



if $\tan x = -1$

if $x = \dfrac{3\pi}{4}$ (QUAD. 2) $\qquad 0 < \dfrac{3\pi}{2} < 2\pi$

or $x = \dfrac{7\pi}{4}$ (QUAD. 4) $\qquad 0 < \dfrac{7\pi}{4} < 2\pi$



$f(x)$ INCREASING $\quad (0, \frac{3\pi}{4}) \quad (\frac{7\pi}{4}, 2\pi)$

DECREASING $\quad (\frac{3\pi}{4}, \frac{7\pi}{4})$