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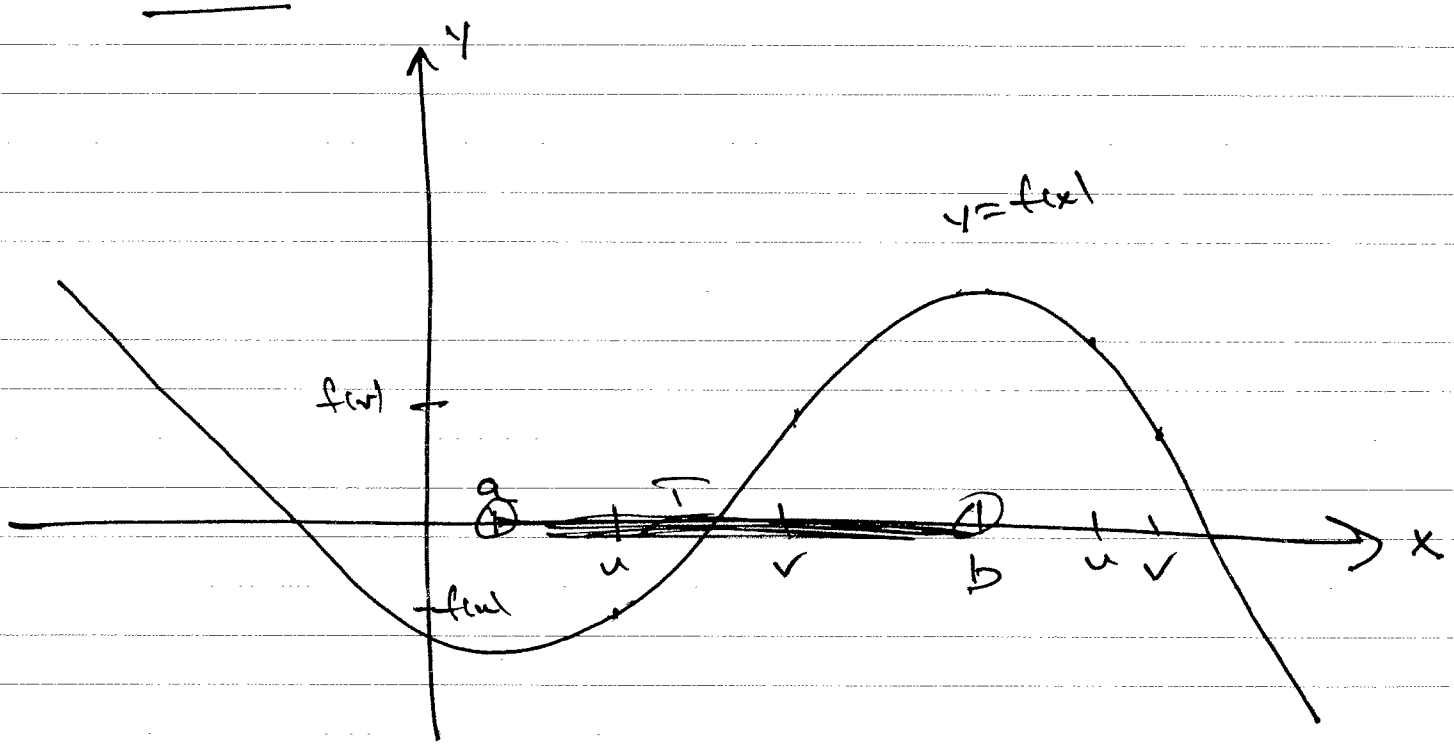
§4.4 MONOTONIC FUNCTIONS
§4.6 LOCAL EXTREMA
} RELATED

EXAMPLE



NOTE: $f(x)$ IS DECREASING ON INTERVALS $(-\infty, a)$ AND (b, ∞).

$f(x)$ IS INCREASING ON $(a, b) = I$

$(-\infty, a)$ (a, b) (b, ∞) ARE THE INTERVALS OF MONOTONICITY OF THE FUNCTION $f(x)$

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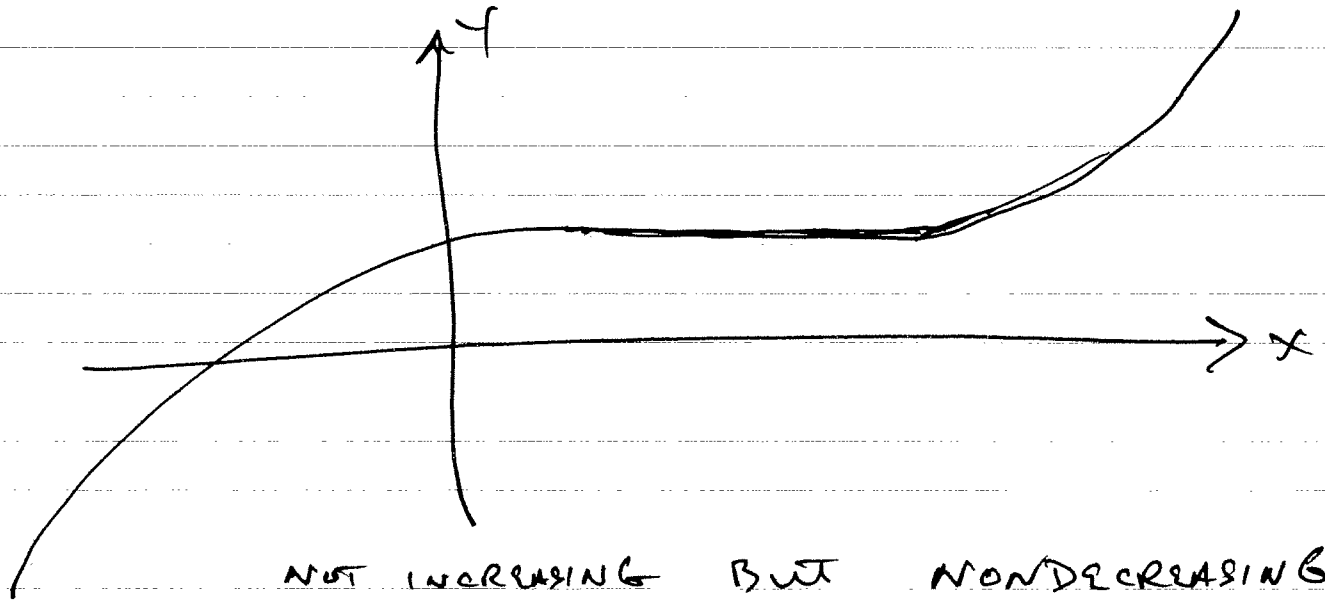
DEF'N

$f(x)$ IS INCREASING ON THE OPEN INTERVAL, I, IF FOR ANY TWO NUMBERS u, v IN I WITH $u < v$, IT IS ALWAYS THE CASE THAT $f(u) < f(v)$

$f(x)$ IS DECREASING ON THE OPEN INTERVAL I IF FOR ANY u, v IN I IT IS ALWAYS THE CASE THAT $f(u) > f(v)$

$f(x)$ IS NONDECREASING ON I IF IT IS INCREASING EXCEPT FOR POSSIBLE "HORIZONTAL PIECES".



NOT INCREASING BUT NONDECREASING.

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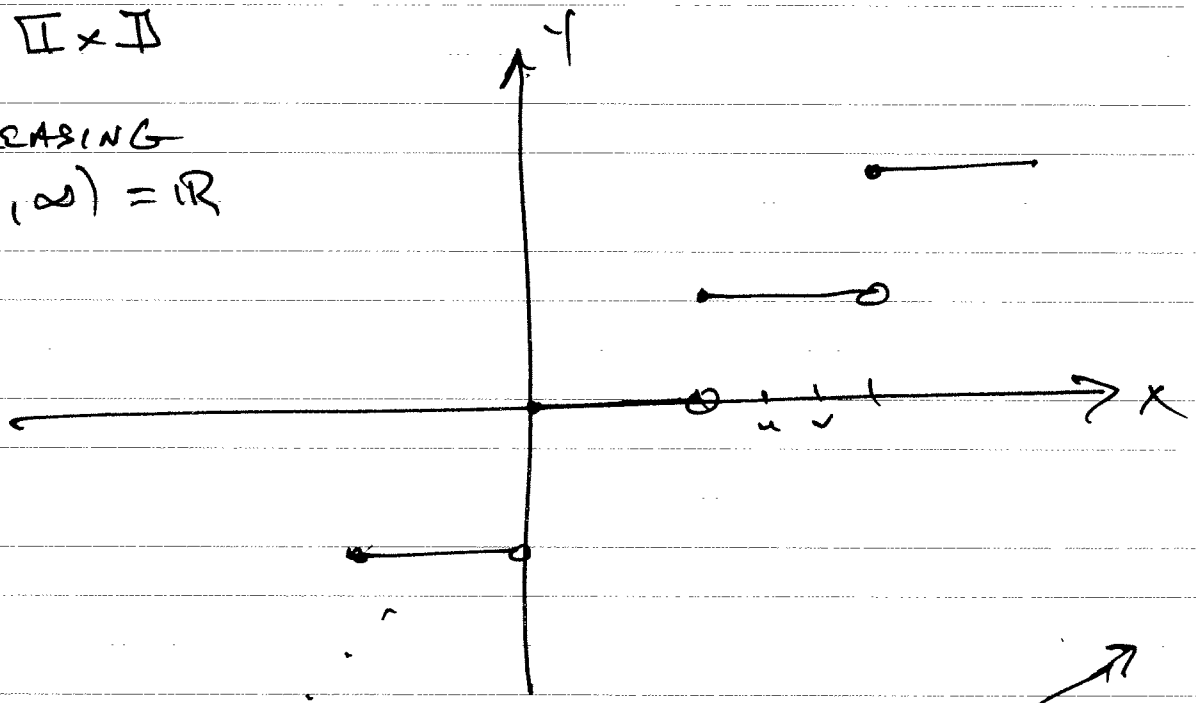
$f(x)$ IS NONINCREASING IF IT IS ~~[illegible]~~ DECREASING EXCEPT FOR HORIZONTAL PIECES.

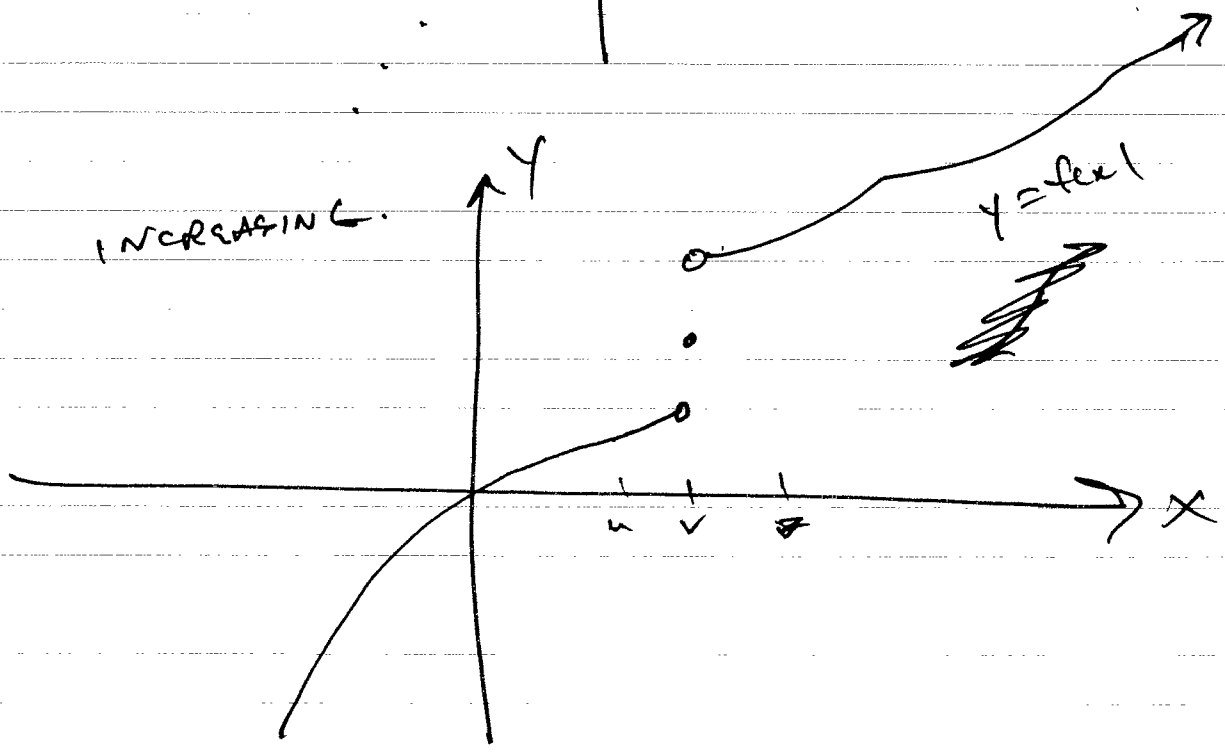
Ex (1) $y = \mathbb{I} \times \mathbb{I}$

NONDECREASING ON $(-\infty, \infty) = \mathbb{R}$



(2)

INCREASING.



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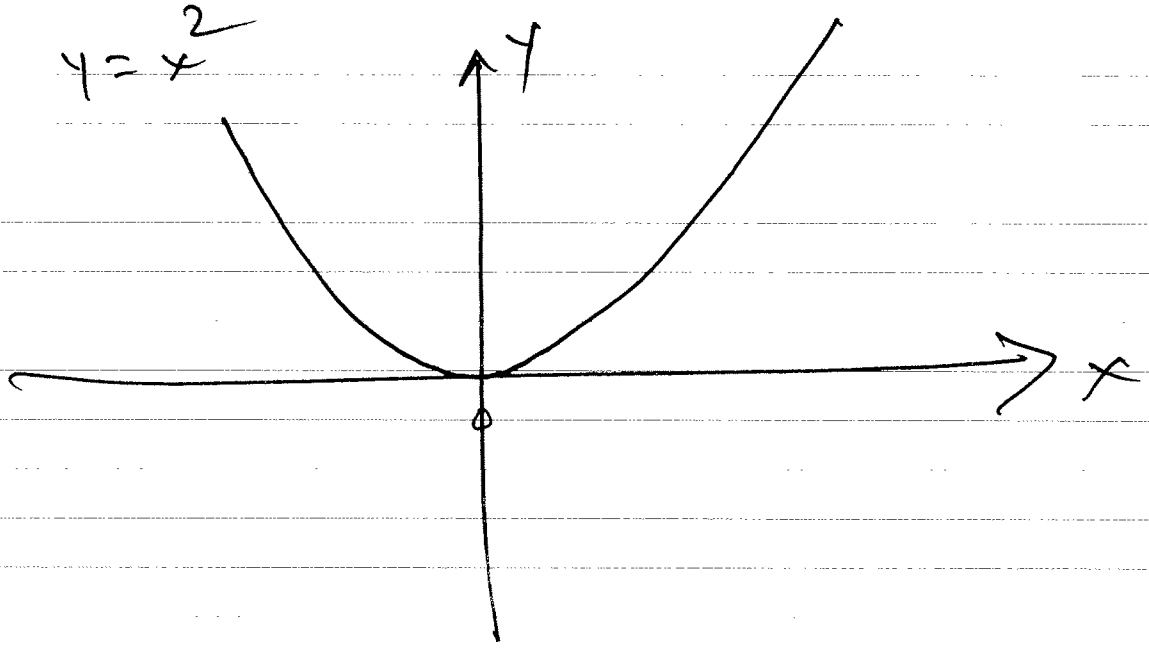
(3) $y = x^2$



INCREASING ON $(0, \infty)$

DECREASING ON $(-\infty, 0)$

NOT MONOTONIC ON $\mathbb{R}$.

A FUNCTION $f(x)$ IS MONOTONIC ON I IF IT HAS ONE OF THE FOUR TYPES OF MONOTONICITY.

"PROBLEM": HOW TO TEST A FUNCTION FOR MONOTONICITY?

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MONOTONICITY RULE

(i) IF $f'(x) > 0$ FOR ALL x IN THE OPEN INTERVAL, I, THEN $f(x)$ IS INCREASING ON I.

(ii) IF $f'(x) < 0$ FOR ALL x IN THE OPEN INTERVAL, I, THEN $f(x)$ IS DECREASING ON I.

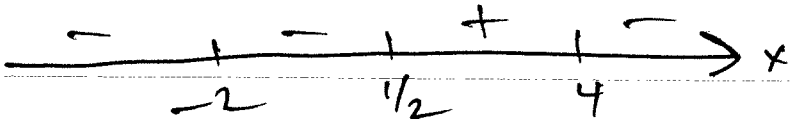
$f'(x) > 0$,	$f(x)$ ↗
$f'(x) < 0$,	$f(x)$ ↘

MONOTONICITY RULE.

EXAMPLE IF $f'(x) = \dfrac{-3(x-4)^3(x+2)^2}{2x-1}$,

FIND THE INTERVALS OF MONOTONICITY OF $f(x)$.

SOLN f'



$f(x)$ IS DECREASING ON $(-\infty, \frac{1}{2}]$ AND ON $[4, \infty)$.

$f(x)$ IS INCREASING ON $(\frac{1}{2}, 4)$.

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DEF'N $x = c$ IS A CRITICAL NUMBER OF $f(x)$

IF EITHER (i) $f'(c) = 0$
OR (ii) $f'(c)$ IS UNDEFINED.