Department of Mathematics and Statistics
Memorial University of Newfoundland

NOV 27/09.

INDEFINITE INTEGRALS

1. $\int \left(\sqrt[5]{x^4} + \frac{1}{x^6} \right) dx$

SOL'N TYPE II

$$\int \left(\sqrt[5]{x^4} + \frac{1}{x^6} \right) dx = \int \left(x^{\frac{4}{5}} + x^{-6} \right) dx$$

$$= \frac{x^{9/5}}{9/5} + \frac{x^{-5}}{-5} + C$$

$$= \frac{5}{9} x^{9/5} - \frac{1}{5} x^{-5} + C$$

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2. $\int \frac{2}{1+3x} dx$

$$\boxed{\int \frac{1}{mx+b} dx = \frac{1}{m} \ln|mx+b| + C}$$

SOL'N TYPE II

$$\int \frac{2}{1+3x} dx = 2 \int \frac{1}{1+3x} dx$$

$$= 2 \, \frac{\ln|1+3x|}{3} + C$$

$$= \frac{2}{3} \ln|1+3x| + C$$

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3. $\int \sec^2(1-2x)\,dx$

SOL'N TYPE I.

$$\int \sec^2(1-2x)\,dx = -\frac{1}{2}\tan(1-2x) + C$$

4. $\int \frac{x^2-4}{2-x}\,dx$

SOL'N TYPE III

$$\int \frac{x^2-4}{2-x}\,dx = \int \frac{\overset{-1}{(x-2)}(x+2)}{2-x}\,dx$$

$$= -\int x+2\,dx$$

$$= -\frac{1}{2}x^2 - 2x + C$$

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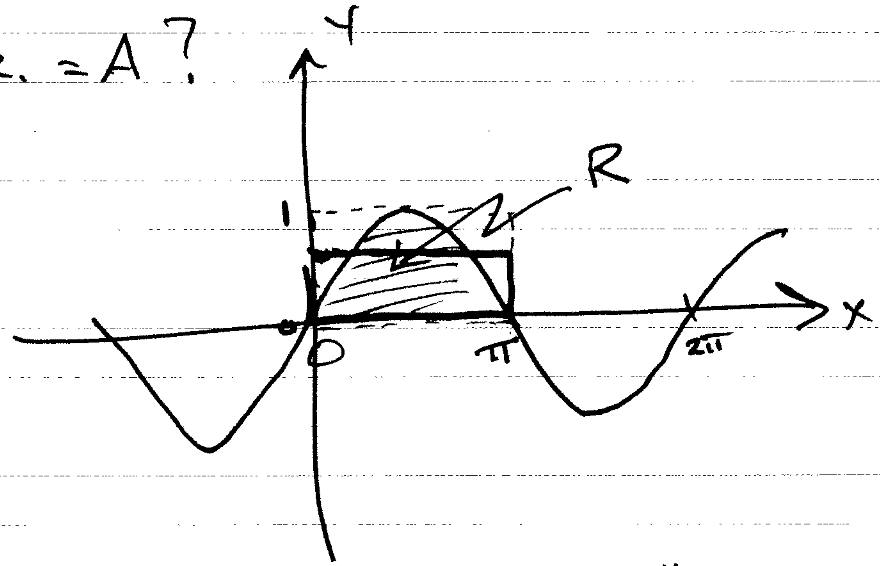
§ 5.4 DEFINITE INTEGRAL

PROBLEM FIND THE AREA BETWEEN ONE ARCH OF THE SINE FUNCTION $f(x) = \sin x$ AND THE X-AXIS.

SOL'N ? AREA OF R = A ?

$0 < A < \pi$



WHAT IF THERE IS A "MAGIC RECTANGLE" WHOSE AREA IS EXACTLY THE SAME AS THE AREA OF R?

WHAT IS THE HEIGHT y^* OF THE MAGIC RECTANGLE?

$$y^* \approx \frac{\sin\frac{\pi}{6} + \sin\frac{\pi}{3} + \sin\frac{\pi}{2} + \sin\frac{2\pi}{3} + \sin\frac{5\pi}{6} + \sin\pi}{6}$$

$$= \frac{\frac{1}{2} + \frac{\sqrt{3}}{2} + 1 + \frac{\sqrt{3}}{2} + \frac{1}{2} + 0}{6}$$

$$= \frac{2+\sqrt{3}}{6} \approx 0.62$$

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USE MORE POINTS!

$$\lim_{n\to\infty} \frac{f(x_1) + f(x_2) + \cdots + f(x_n)}{n} = y^*$$

IF THE LIMIT EXISTS.

$$\text{AREA OF } A = \pi \cdot \left\{ \lim_{n\to\infty} \frac{f(x_1) + f(x_2) + \cdots + f(x_n)}{n} \right\}$$

IF THE LIMIT EXISTS,

FOR OUR PROBLEM $y^* = \frac{2}{\pi}$.

$\Rightarrow$ AREA UNDER ONE ARCH OF THE SINE F'N IS $A = \pi \cdot \frac{2}{\pi} = 2$ SQ UNITS

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DEF'N THE AVERAGE Y-VALUE OF THE FUNCTION $f(x)$ ON THE INTERVAL $[a,b]$, DENOTED BY y^* OR $AVE(f(x),[a,b])$, IS DEFINED BY

$$AVE(f(x),[a,b]) = \lim_{n\to\infty} \frac{f(x_1)+f(x_2)+\cdots+f(x_n)}{n}$$

IF THE LIMIT EXISTS.

ASSUME THE x_k's ARE EQUALLY SPACED.

DEF'N THE DEFINITE INTEGRAL OF $f(x)$ OVER $[a,b]$ DENOTED BY $\int_a^b f(x)\,dx$, IS DEFINED BY

$$\int_a^b f(x)\,dx = (b-a)\,AVE(f(x),[a,b])$$

IF THE UNDERLYING LIMIT IN $AVE(f(x),[a,b])$ EXISTS.