Department of Mathematics and Statistics
Memorial University of Newfoundland

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§ 5.1
→ 5.2 RULES (SHEET)

$\int f(x)\,dx$ MEANS

(1) FIND $F(x)$ SUCH THAT $F'(x) = f(x)$
ANTIDERIVATIVE

(2) ADD AN UNSPECIFIED CONSTANT OF INTEGRATION
C

$$\int f(x)\,dx = F(x) + C$$

EXAMPLES FIND $\int f(x)\,dx$.

1) $\int x^{1.8}\,dx = \frac{x^{2.8}}{2.8} + C$

2) $\int \frac{1}{x^2}\,dx = \int x^{-2}\,dx = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$

3) $\int \sin x\,dx = -\cos x + C$

4) $\int \sec^2 x\,dx = \tan x + C$

5) $\int \frac{1}{x}\,dx = \int x^{-1}\,dx = \ln|x| + C$

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6. $\int f'(x)\,dx = f(x) + C$

7. $\int f''(x)\,dx = f'(x) + C$

8. $\int v(t)\,dt = s(t) + C$

9. $\int a(t)\,dt = v(t) + C$

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7. WHAT ABOUT INTEGRALS LIKE:

1) $\int (3x^2 - 2x + 1)\,dx$

Rule 2) $\int -7\sin x\,dx$

3) $\int x^2(1+x)\,dx$

4) $\int \cos(3x+1)\,dx$

NOT IN TABLE 1.!!

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TYPE I, II, III INDEFINITE INTEGRALS

TYPE I $\int f(x)\,dx$ OR $\int f(mx+b)\,dx$ IS TYPE I IF $\int f(x)\,dx$ APPEARS IN TABLE 1 (TABLE 1 OR RULE 3)

TYPE II AN INTEGRAL IS TYPE II IF RULES 1 AND/OR 2 APPLY. (RULES 1 OR 2)

TYPE III NO RULE APPLIES. BUT, SOME LEGAL ALGEBRA CONVERTS THE INTEGRAL TO A TYPE I OR TYPE 2.

EXAMPLES

1) $\int (3x^2 - 2x + 1)\,dx = \int 3x^2\,dx - \int 2x\,dx + \int 1\,dx$

TYPE II

$= 3\int x^2\,dx - 2\int x\,dx + \int 1\,dx$

$= 3 \cdot \frac{1}{3}x^3 - 2\frac{1}{2}x^2 + x + C$

$= x^3 - x^2 + x + C$

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(2) $\int -7 \sin x \, dx$ Table 1.

SOLN TYPE II

$$\int -7 \sin x \, dx = -7 \int \sin x \, dx$$

$$= -7(-\cos x) + C$$

$$= 7 \cos x + C$$

(3) $\int x^2 (1+x) \, dx$

SOLN TYPE III "ALGEBRA"

$$\int x^2 (1+x) \, dx = \int (x^2 + x^3) \, dx \quad \text{TYPE II}$$

$$= \frac{1}{3} x^3 + \frac{1}{4} x^4 + C$$

(4) $\int \cos(3x+1) \, dx$

SOLN TYPE I.

$$\int \cos(3x+1) \, dx = \frac{1}{3} \sin(3x+1) + C \quad \text{RULE 3}, \; m=3$$

Guess & Check $\int \cos(3x+1) \, dx = \frac{1}{3} \sin(3x+1) + C$

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METHOD 3 $\int \cos(3x+1)\,dx$

SOL'N SUBSTITUTION METHOD

$u = 3x+1$
$du = 3\,dx$

$$\int \cos(3x+1)\,dx = \int \cos u \left(\frac{1}{3}\,du\right)$$

$$= \frac{1}{3}\int \cos u\,du$$

$$= \frac{1}{3}\sin u + C$$

$$= \frac{1}{3}\sin(3x+1) + C$$

(5) $\int e^{-2x+1}\,dx$

SOL'N TYPE I $\int e^x\,dx = e^x + C$

$$\int e^{-2x+1}\,dx = -\frac{1}{2}e^{-2x+1} + C$$

$m = -2$

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4. $\int \sin x \cos x \, dx$

$\sin 2\theta = 2 \sin\theta \cos\theta$

SIN TYPE III

$$\int \sin x \cos x \, dx = \int \frac{1}{2} \sin 2x \, dx \quad \text{TYPE II}$$

$$= \frac{1}{2} \int \sin 2x \, dx \quad \text{TYPE I.}$$

$$= -\frac{\cos(2x)}{4} + C$$