Nov 27/09

Procedure for Max–Min Problems

1. Identify the quantity, Q, to be maximized or minimized.

2. Make a diagram assigning any necessary variables.

3. Write down an equation for Q.

4. Write down any *constraint equations.* Use these to produce a normal looking expression $Q = Q(x)$. Note any logical restriction on x.

5. Find $Q'(x)$ and factor it. Get the sign pattern of Q'. Note the absolute extremum.

6. Write down the required answer in a proper sentence.

A woman with 300 metres of fencing material wishes to enclose a rectangular area along the bank of a straight river. No fence is required along the river. What dimensions should she use to maximize the enclosed area?

SOL'N LET A REPRESENT THE AREA TO BE MAXIMIZED.

$A = xy$

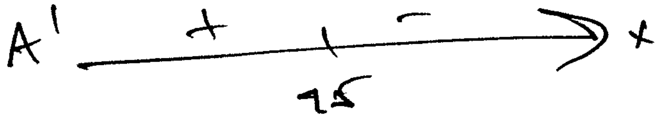
$A = x(300 - 2x)$

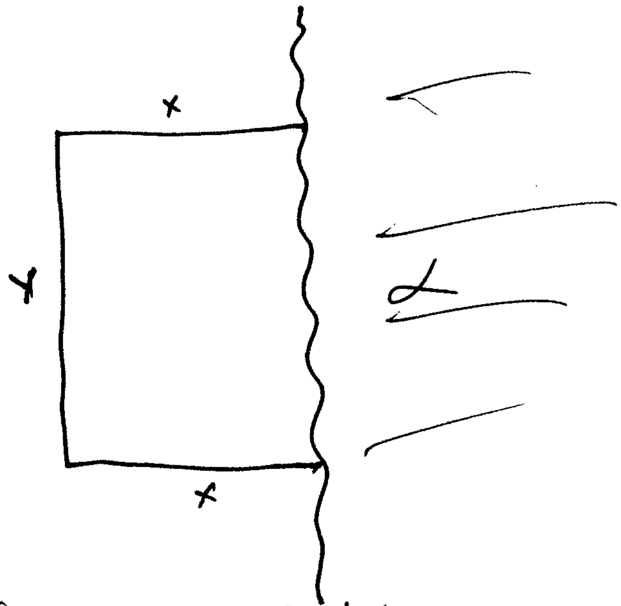
$0 \le x \le 150$

$A = A(x) = 300x - 2x^2$

$A' = 300 - 4x$

$= 4(75 - x)$





CONSTRAINT EQ'N

$2x + y = 300$

$y = 300 - 2x$

$\boxed{0 \le x \le 150}$

FROM THE CONSTRAINT, WHEN $x = 75$, $y = 150$

DIMENSIONS FOR MAX AREA

ARE 75m BY 150m.

A closed box with a square base is to have a volume of 250 cubic metres. The material for the top and bottom of the box costs \$2 per square metre and the material for the sides costs \$1 per square metre. Can the box be constructed for less than \$300?

SOL'N LET C REPRESENT THE COST OF THE BOX TO BE MINIMIZED.

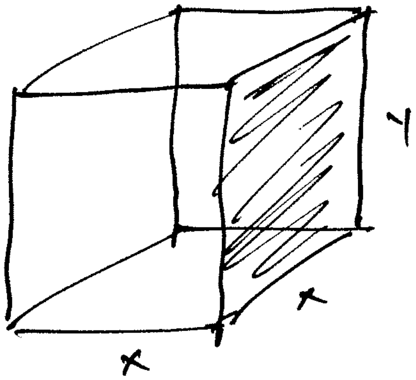
$$C = C_{TB} + C_{4S}$$

$$= \$2\,(2x^2) + \$1\,(4xy)$$

$$C = 4x^2 + 4xy.$$

$$C = 4x^2 + 4x\left(\frac{250}{x^2}\right)$$

$$\boxed{C = 4x^2 + \frac{4\cdot 250}{x}} \quad \boxed{x>0}$$



CONSTRAINT

$$x\cdot x\cdot y = 250$$

$$x^2 y = 250$$

$$y = \frac{250}{x^2}$$

$$x > 0$$

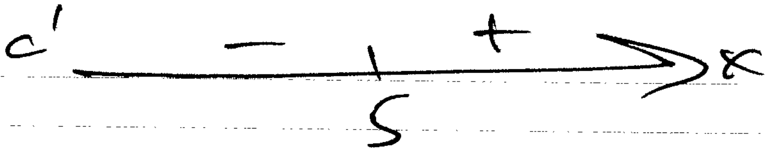
$$(x^{-1})' = -x^{-2} = -\frac{1}{x^2}$$

$$c' = 8x - \frac{4\cdot 250}{x^2}$$

$$= 8x - 4\cdot 250\, x^{-2}$$

$$= x^{-2}(8x^3 - 4\cdot 250)$$

$$= \frac{8(x^3 - 125)}{x^2}$$



$$c(5) = 4\cdot 25 + \frac{4\cdot 250}{5} = 100 + 200 = 300$$

NO! MIN. COST IS \$300.

$$(x-h)^2+(y-k)^2=r^2$$

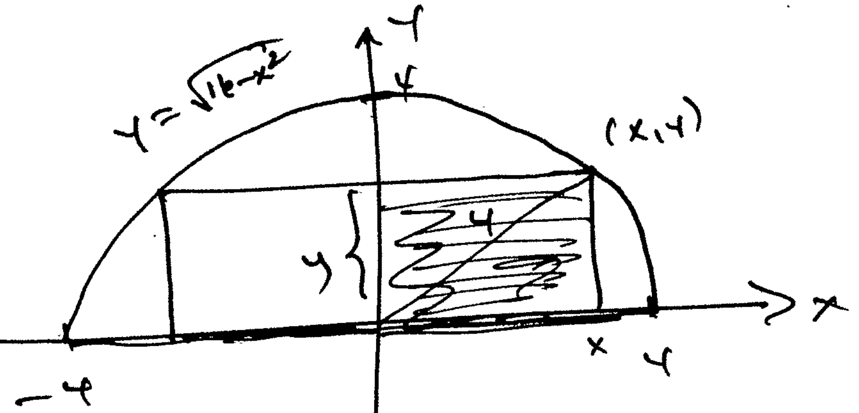
3. Find the dimensions of the rectangle of maximum area that can be inscribed in a semicircle of radius 4 if two vertices lie on the diameter.

SOL'N LET A REPRESENT THE AREA TO BE MAXIMIZED.

$A = 2A_{\text{QUAD } 1}$

$A = 2xy$

$$\boxed{A = 2x\sqrt{16-x^2}}$$



$$\boxed{x^2+y^2=4^2}$$

$y^2=16-x^2$

$y=\pm\sqrt{16-x^2}$

$0\le x\le 4$

$y=\sqrt{16-x^2}$

$A(x) = 2x(16-x^2)^{1/2}$

$A' = \cancel{2}x \cdot \frac{1}{\cancel{2}}(16-x^2)^{-1/2}(-2x) + 2\cdot(16-x^2)^{1/2}$

$(2\sqrt{2})+2\sqrt{2}$
$=4\sqrt{2}$

$= 2(16-x^2)^{-1/2}\left(-x^2+16-x^2\right)$

$= \dfrac{2(16-2x^2)}{\sqrt{16-x^2}} = \dfrac{4(8-x^2)}{\sqrt{16-x^2}}$

A' + | −

$\sqrt{8}=2\sqrt{2}$

$$y = \sqrt{16 - (\sqrt{8})^2} = \sqrt{8} = 2\sqrt{2}$$

DIMENSIONS OF LARGEST RECTANGLE ARE $4\sqrt{2}$ BY $2\sqrt{2}$.