Department of Mathematics and Statistics
Memorial University of Newfoundland

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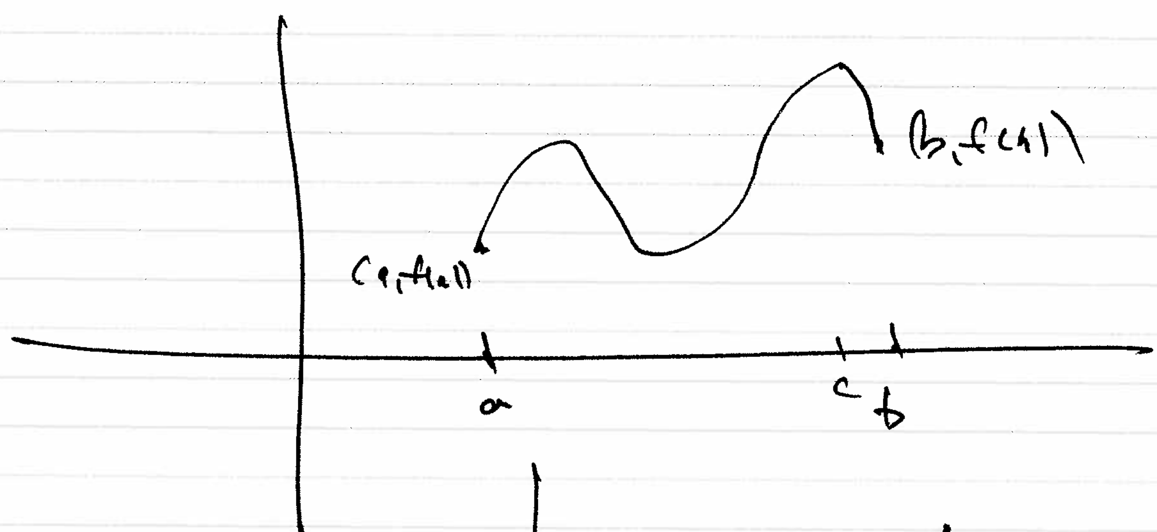
§4.11 Absolute Extrema
§4.12 Applied Max/Min Problems

PROBLEM: FIND ANY ABSOLUTE EXTREMA OF $f(x)$ ON ITS DOMAIN, D_f.

POSSIBILITIES: A GIVEN $f(x)$ WITH DOMAIN D_f MAY HAVE 0, 1, OR 2 ABSOLUTE ~~EXTREMA~~ EXTREMA.

ONE THEOREM THAT GUARANTEES THE EXISTENCE OF BOTH ABSOLUTE EXTREMA IS THE EXTREME VALUE THEOREM p 123.

EVT



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"FINDING ABS. EXTREMA GIVEN $f(x)$ AND D_f."

<u>CASE 1</u> $f \in C[a,b]$ $f(x)$ IS CONTINUOUS ON $[a,b]$.

<u>CASE 2</u> ANYTHING BUT CASE 1.

<u>CONSIDER CASE 1</u> ABS. EXTREMA EXIST BY EVT

ABS. EXT. MUST OCCUR AT A CRITICAL NUMBER <u>OR</u> AT AN ENDPOINT (a OR b).

<u>PROCEDURE FOR CASE 1</u> $f \in C[a,b]$.

1) FIND f' AND [redacted] FACTOR IT.

2) IF THE CRITICAL NUMBERS ARE $c_1, c_2, \ldots, c_n$

FILL IN THE TABLE OF VALUES

x	a	c_1	c_2	-	c_n	b
$f(x)$						

3) LARGEST Y-VALUE IS ABS. MAX
SMALLEST Y-VALUE IS ABS. MIN.

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EXAMPLE FIND ANY ABSOLUTE EXTREMA OF

$f(x) = x^3 + 3x^2 - 1$ ON $D_f = [-4, 1]$.

SOL'N CASE 1 APPLIES

$f'(x) = 3x^2 + 6x = 3x(x+2)$ CRIT. #'s $0, -2$

x	-4	-2	0	1
y	-17	3	-1	3

$f(-4) = -64 + 48 - 1$
$= -17$
$f(-2) = -8 + 12 - 1$

ABS MAX IS 3. (IT OCCURS AT BOTH $x = -2$, $x = 1$)
ABS MIN IS -17 (IT OCCURS AT LEFT ENDPOINT $x = -4$.

CASE 2 (ANYTHING BUT CASE 1).

PROCEDURE FOR CASE 2

1) FIND f' AND FACTOR IT.

2) GET THE SIGN PATTERN OF f'.

3) USING THE SIGN PATTERN OF f' AND/OR A VERY ROUGH SKETCH OF THE GRAPH OF $f(x)$, IDENTIFY ANY ABS. EXTREMA.

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ABS. EXTREMA

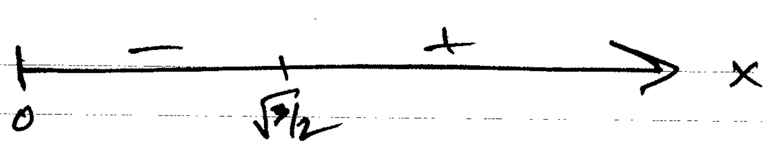
EXAMPLE 2 $f(x) = x^4 - 3x^2 - 5$ ON $D_f = [0, \infty)$

SOL'N CASE 2 D_f IS UNBOUNDED.

(i) $f'(x) = 4x^3 - 6x = 2x(2x^2 - 3)$

CRIT #'S: $x = 0$, $x = \pm\sqrt{3/2}$

(ii) f'



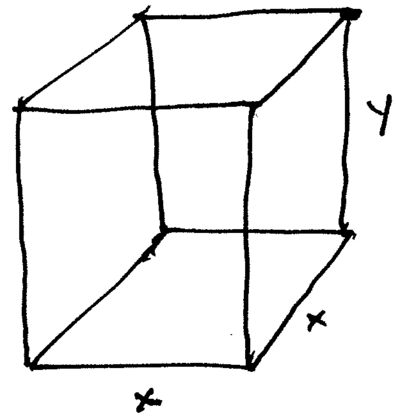
FROM THE SIGN PATTERN, $x = \sqrt{3/2}$ GIVES THE ABS. MIN.

$$f(\sqrt{3/2}) = \left(\sqrt{\tfrac{3}{2}}\right)^4 - 3\left(\sqrt{\tfrac{3}{2}}\right)^2 - 5$$

$$= \frac{9}{4} - \frac{9}{2} - 5 = \frac{9 - 18 - 20}{4} = -\frac{29}{4}$$

SINCE $\lim\limits_{x \to \infty} (x^4 - 3x^2 - 5) = \infty$ THERE CANNOT BE AN ABS. MAX.

3. **Parcel Post regulations require that a rectangular parcel be such that the sum of the height and the distance around the base not exceed six feet. What is the maximum volume of a parcel with a square base?**



$$\boxed{V = l \cdot w \cdot h}$$

SOL'N

LET V REPRESENT THE VOLUME TO BE MAXIMIZED.

$\Rightarrow V = x^2 y$

CONSTRAINT EQUATION

$4x + y = 6$

FROM THE CONSTRAINT EQ'N

$y = 6 - 4x = 2(3 - 2x)$

$D_V = [0, \frac{3}{2}]$ ← DOMAIN OF V.

$V = V(x)$

$V = x^2(6 - 4x)$

$= 6x^2 - 4x^3$

$V' = 12x - 12x^2$

$= 12x(1 - x)$

V': + on $(0, 1)$, − after 1

ABS. MAX OCCURS WHEN $x = 1$.

MAX VOLUME IS $V(1) = 6 - 4 = 2$ cu. ft.