

Sp 4.8 Curve Sketching

Example 9

#3 $f(x) = x^{\frac{2}{5}}(x-2)$ $D_f = \mathbb{R}$

$$f'(x) = x^{\frac{2}{5}} + \frac{2}{5}x^{-\frac{3}{5}}(x-2) = \frac{1}{5}x^{-\frac{1}{5}}(5x + 2(x-2))$$

$$= \frac{7x-4}{5x^{\frac{3}{5}}} \quad (\frac{5x^{\frac{3}{5}}}{x})' = 3x^{\frac{2}{5}}$$

$$f''(x) = \frac{5x^{\frac{3}{5}} \cdot 7 - (7x-4) \cdot 3x^{-\frac{4}{5}}}{(5x^{\frac{3}{5}})^2} \quad \frac{7}{5}$$

$$= \frac{x^{-\frac{2}{5}}(35x - 3(7x-4))}{25x^{\frac{6}{5}}} \quad (-1) = -1$$

$$= \frac{35x - 21x + 12}{25x^{\frac{8}{5}}} = \frac{2(7x+6)}{25x^{\frac{8}{5}}}$$

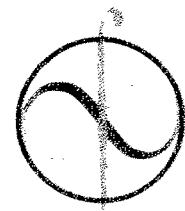
$$f' \begin{array}{c} + \\ \hline 0 \\ - \\ 4/7 \\ + \end{array} \rightarrow x$$

$$f'' \begin{array}{c} + \\ -6/7 \\ 0 \\ + \\ + \end{array} \rightarrow x$$

loc max: $x=0$ loc min $x=4/7$

Point of Inflection: $x=-6/7$

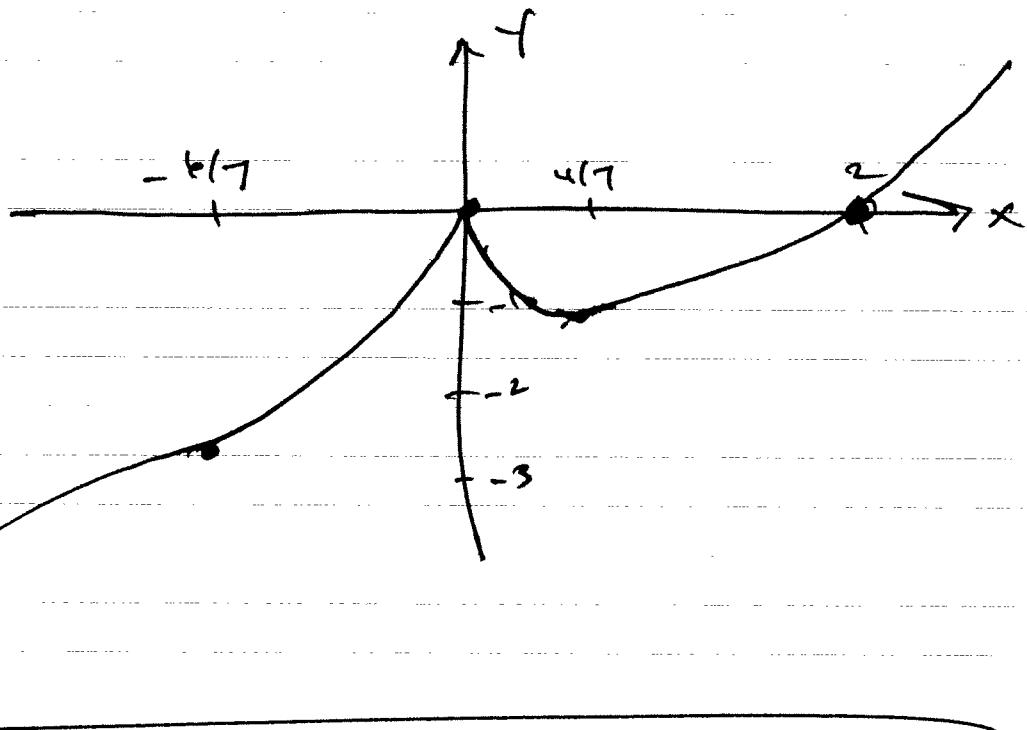
(2)



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$$f(x) = x^2 e^{-x} \quad \text{Intercepts } x=0, x=2$$

x	y
-4/7	-2.7
0	0
4/7	-1.1
2	0



$$\textcircled{4} \quad f(x) = x^2 e^{-x} \quad (\text{or } f(x) = \frac{x^2}{e^x})$$

$$f'(x) = -x^2 e^{-x} + 2x e^{-x} = x e^{-x} (2-x) = \frac{x(2-x)}{e^x}$$

$$f''(x) = +x^2 e^{-x} - 2x e^{-x} - 2x e^{-x} + 2 e^{-x} \\ = \frac{-x^2 e^{-x} - 2x e^{-x} + 2 e^{-x}}{e^{-x}} = \frac{-x^2 - 2x + 2}{e^{-x}} = \frac{x^2 - 4x + 2}{e^{-x}}$$

$$f' \quad \begin{array}{ccccccc} - & + & & + & - & & \end{array} \rightarrow x \quad \begin{matrix} 0 \\ 2 \end{matrix}$$

$$= \frac{x^2 - 4x + 2}{e^{-x}}$$

$$f'' \quad \begin{array}{ccccc} + & & - & + & + \end{array} \rightarrow x \quad \begin{matrix} 2-\sqrt{2} \\ 2+\sqrt{2} \end{matrix}$$

Hypercritical #'s

$$x = \frac{4 \pm \sqrt{16-8}}{2}$$

$$= \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

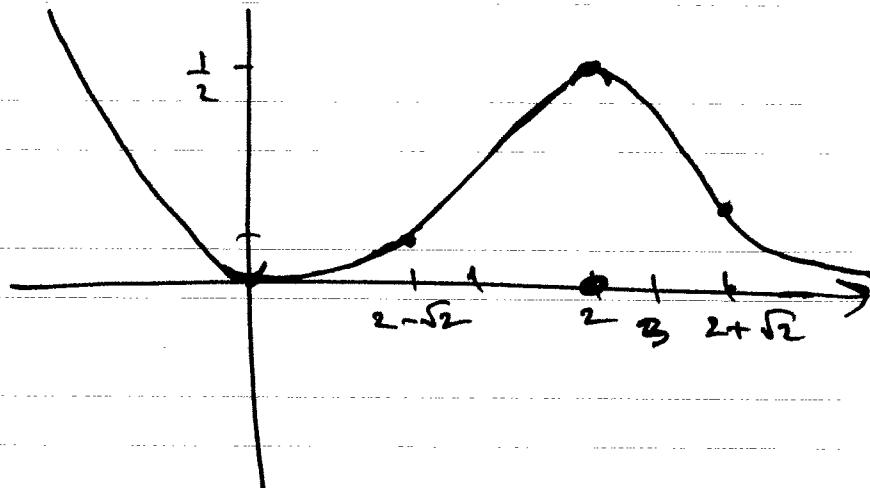


Integrates $x \geq 0$

$$\lim_{x \rightarrow \infty} \frac{x^2}{e^x} = 0$$

$y=0$
Horiz. Asym.

x	y
0	0
$2 - \sqrt{2}$	≈ 0.19
2	$\frac{4}{e^2} \approx 0.54$
$2 + \sqrt{2}$	≈ 0.38



Local min $x=0$

Local max $x=2$

Flex Points: $x = 2 \pm \sqrt{2}$.

§ 4.11 Absolute Extrema

4.12 Applied Max/min

§ 4.11

GIVEN: A function $f(x)$ with domain D_f .

RECALL $f(c)$ is the ABSOLUTE maximum (minimum)
OF $f(x)$ IF $f(c) \geq f(x)$ ($f(c) \leq f(x)$) FOR
ALL x IN D_f .