Nov 19/09.

§4.8 Curve Sketching

Examples

#3 $\boxed{f(x) = x^{2/5}(x-2)}$ $\mathcal{D}_f = \mathbb{R}$

$$f'(x) = x^{2/5} + \frac{2}{5}x^{-3/5}(x-2) = \frac{1}{5}x^{-3/5}\left(5x + 2(x-2)\right)$$

$$= \boxed{\frac{7x-4}{5x^{3/5}}}$$

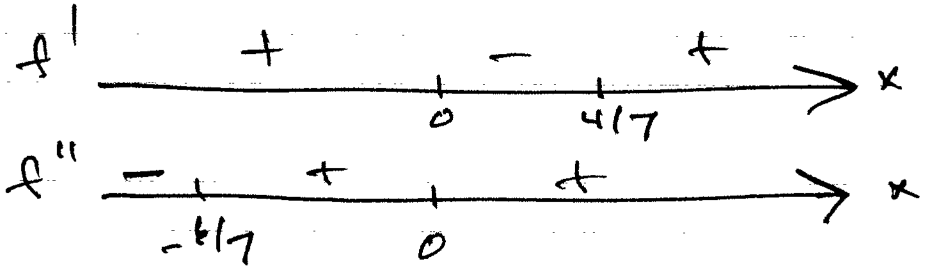
$$(5x^{3/5})' = 3x^{-2/5}$$

$$f''(x) = \frac{5x^{3/5}\cdot 7 - (7x-4)\,3x^{-2/5}}{(5x^{3/5})^2}$$

$$= \frac{x^{-2/5}\left(35x - 3(7x-4)\right)}{25x^{6/5}}$$

$$(-1)^{3/5} = -1$$

$$= \frac{35x - 21x + 12}{25x^{8/5}} = \boxed{\frac{2(7x+6)}{25x^{8/5}}}$$

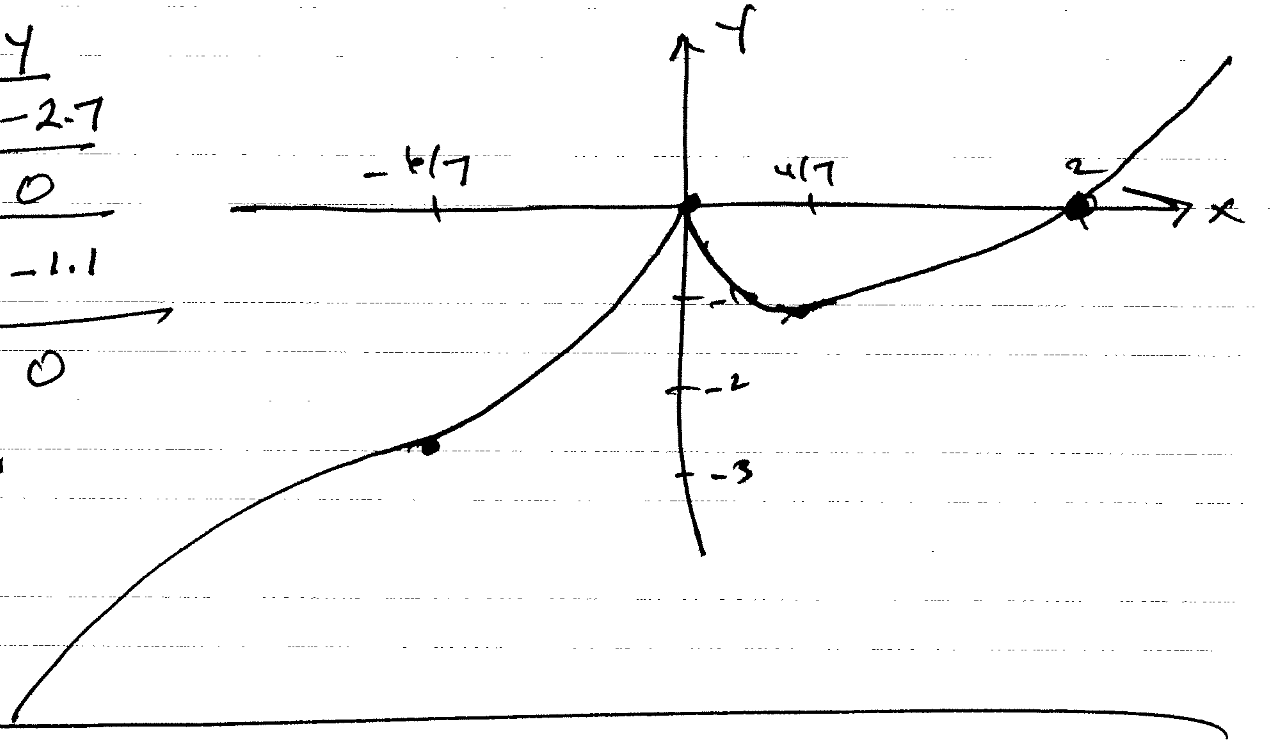


Loc max: $x=0$ Loc. min $x = 4/7$

Point of inflection: $x = -6/7$

$f(x) = x^{2/5}(x-2)$ Intercepts $x=0, x=2$

x	y
$-4/7$	-2.7
0	0
$\frac{4}{7}$	-1.1
2	0



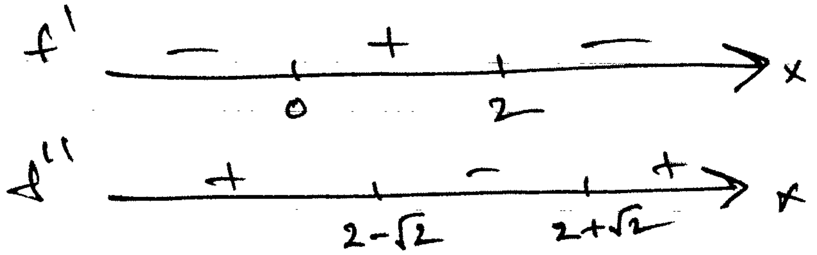
④ $f(x) = x^2 e^{-x}$ (or $f(x) = \frac{x^2}{e^x}$)

$$f'(x) = -x^2 e^{-x} + 2x e^{-x} = x e^{-x}(2-x) = \boxed{\frac{x(2-x)}{e^x}}$$

$$f''(x) = +x^2 e^{-x} - 2x e^{-x} - 2x e^{-x} + 2 e^{-x}$$

$$= e^{-x}(x^2 - 2x - 2x + 2) = e^{-x}(x^2 - 4x + 2)$$

$$= \boxed{\frac{x^2 - 4x + 2}{e^x}}$$



Hypercritical #'s

$$x = \frac{4 \pm \sqrt{16-8}}{2}$$

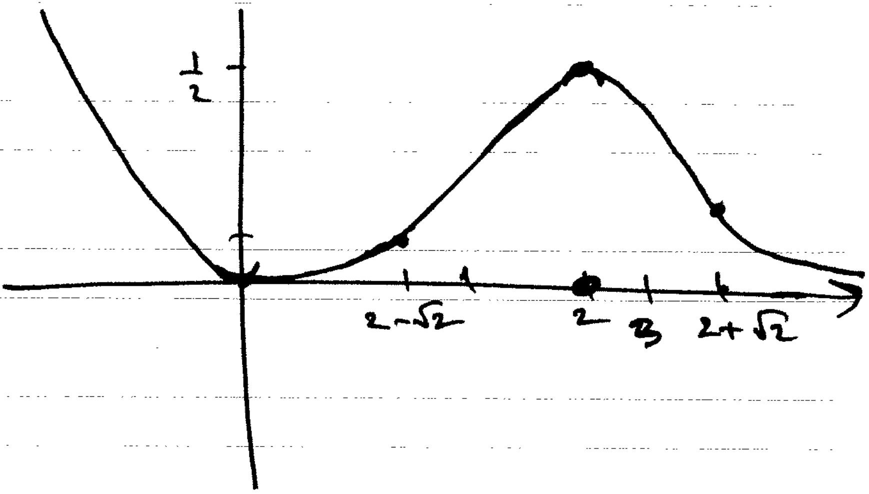
$$= \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

Intercepts $x=0$

$$\boxed{\lim_{x\to\infty} \frac{x^2}{e^x} = 0}$$

$y=0$
Horiz. Asym.

x	y
0	0
$2-\sqrt{2}$	≈ 0.19
2	$\frac{4}{e^2} \approx 0.54$
$2+\sqrt{2}$	≈ 0.38



Local Min $x=0$
Local Max $x=2$
Flex Points: $x=2\pm\sqrt{2}$.

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§ 4.11 Absolute Extrema
4.12 Applied Max/Min

§ 4.11

GIVEN: A FUNCTION $f(x)$ WITH DOMAIN D_f.

RECALL $f(c)$ IS THE ABSOLUTE MAXIMUM (MINIMUM) OF $f(x)$ IF $f(c) \geq f(x)$ $\left(f(c) \leq f(x) \right)$ FOR **ALL** x IN D_f.