Department of Mathematics and Statistics
Memorial University of Newfoundland

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§ 4.4, 4.6 MONOTONICITY, LOCAL EXTREMA

§ 4.7 CONCAVITY, POINTS OF INFLECTION

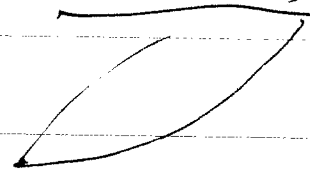
4.8 CURVE SKETCHING

HYPERCRITICAL NUMBER

$f''(c) = 0$ OR $f''(c)$ IS UNDEFINED

$\Longrightarrow$ $x = c$ IS A HYPERCRITICAL NUMBER.

HYPERCRITICAL POINT



$(c, f(c))$

$\uparrow$

Hypercritical Number

and $f(x)$ IS CONTINUOUS AT c.

CONCAVITY RULES

$f''(x) > 0$ ON $I = (a, b)$ THEN $\cup$

$f''(x) < 0$ ON $J = (c, d)$ THEN $\cap$

Def'n $(c, f(c))$ IS AN INFLECTION POINT OF $f(x)$ IF $(c, f(c))$ IS A HYPERCRITICAL POINT AND THE CONCAVITY CHANGES AT c.

"INFLECTION POINT TEST"

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$$f(x) = \frac{x}{1+x^2}$$

PROCEDURE FOR CURVE SKETCHING

1) FIND $f'(x)$ AND FACTOR IT
FIND $f''(x)$ AND FACTOR IT.

$$f'(x) = \frac{(1+x^2)1 - x\,2x}{(1+x^2)^2} = \frac{1-x^2}{(1+x^2)^2}$$

$$f''(x) = \frac{(1+x^2)^2(-2x) - (1-x^2)2(1+x^2)(2x)}{(1+x^2)^4}$$

$$= \frac{-2x\cancel{(1+x^2)}\left((1+x^2) + 2(1-x^2)\right)}{(1+x^2)^{\cancel{4}\,3}} \qquad \cancel{(\sqrt{3}-x)(\sqrt{3}+x)}$$

$$= \frac{-2x(1+x^2)(3-x^2)}{(1+x^2)^4} = \frac{-2x(3-x^2)}{(1+x^2)^3}.$$

2. GET THE SIGN PATTERNS OF f', AND f''.

f': $-$ on $(-\infty,-1)$; $+$ on $(-1,1)$; $-$ on $(1,\infty)$

f'': $-$ on $(-\infty,-\sqrt{3})$; $+$ on $(-\sqrt{3},0)$; $-$ on $(0,\sqrt{3})$; $+$ on $(\sqrt{3},\infty)$

3) APPLY THE FIRST DERIVATIVE TEST TO CRITICAL POINTS.
APPLY THE INFLECTION POINT TEST TO ~~HYPERCRITICAL~~ POINTS.

LOCAL MIN AT $x=-1$. LOCAL MAX $x=1$

INFLECTION POINTS AT $x=\pm\sqrt{3}$, $x=0$.

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4) CHECK FOR INTERCEPTS AND ASYMPTOTES.

$f(x) = \dfrac{x}{1+x^2}$ (0,0) ONLY INTERCEPT.

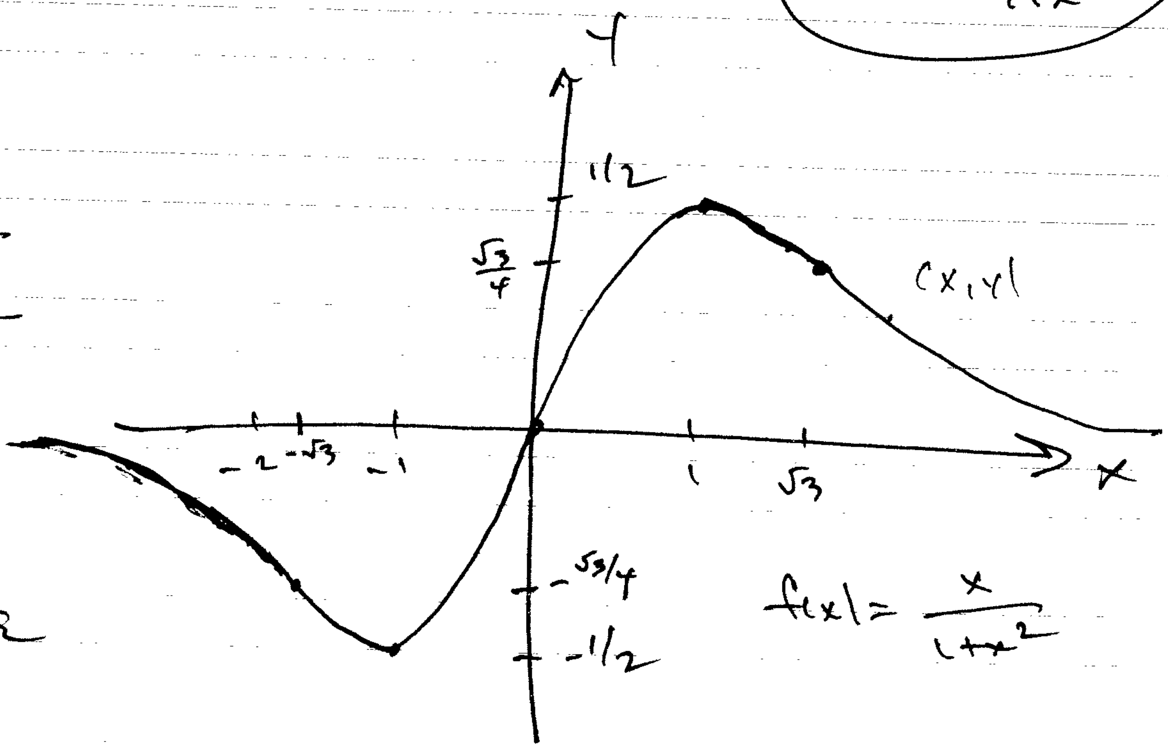
NO VERTICAL ASYMPTOTES $\cancel{\lim_{x \to c^+} f(x) = \infty}$

$\lim_{x \to \infty} f(x) = ?$ $\lim_{x \to -\infty} f(x) = ?$

$\lim_{x \to \infty} \dfrac{x}{1+x^2} = 0$ $y = 0$ IS A HORIZONTAL ASYMPTOTE.

5) MAKE A "SMALL" TABLE OF VALUES. $f(x) = \dfrac{x}{1+x^2}$

x	y
$-\sqrt{3}$	$-\frac{\sqrt{3}}{4}$
-1	$-1/2$
0	0
1	$\frac{1}{2}$
$\sqrt{3}$	$\frac{\sqrt{3}}{4}$



$f(x) = \dfrac{x}{1+x^2}$

6) JOIN THE DOTS.

ODD $f(-x) = -f(x)$

EVEN $f(-x) = f(x)$

Curve Sketching

Sketch the graph of a function $f(x)$, defined and continuous except for $x = \pm 1$, satisfying all of the following.

1. $f(0) = 1$, $f(4) = 0$, $f'(0) = 0$, $f''(0) = 0$

Sign pattern of f'

2. $f'(x) > 0$ on the intervals $(-\infty, -1)$ and $(1, \infty)$

3. $f'(x) < 0$ on the interval $(-1, 1)$

Sign pattern of f''

4. $f''(x) > 0$ on the intervals $(-\infty, -1)$ and $(-1, 0)$

5. $f''(x) < 0$ on the intervals $(0, 1)$ and $(1, \infty)$

Asymptotes

6. $\lim_{x \to \infty} f(x) = 1$ and $\lim_{x \to -\infty} f(x) = 1$ $y=1$ H.A.

7. $\lim_{x \to -1} f(x) = \infty$ and $\lim_{x \to 1} f(x) = -\infty$

$x=-1$ and $x=1$ Even V.A.

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f': $+ \quad (-1) \quad - \quad (0) \quad - \quad (1) \quad +$ $\to x$

f'': $+ \quad (-1) \quad + \quad (0) \quad - \quad (1) \quad -$ $\to x$

x	y
0	1
4	0